



Nơi khởi đầu ước mơ
TUYỂN TẬP TÍCH PHẦN
 (ĐÁP ÁN CHI TIẾT)

BIÊN SOẠN: LƯU HUY THƯỜNG

Toàn bộ tài liệu của thầy ở trang:
<http://www.Luu Huy Thuong.blogspot.com>



HỌ VÀ TÊN:

LỚP :

TRƯỜNG :



HÀ NỘI, 4/2014



TÍCH PHÂN CƠ BẢN

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<http://www.Luu HuyThuong.blogspot.com>

HT 1. Tính các tích phân sau:

$$\text{a) } I_1 = \int_0^1 x^3 dx$$

$$\text{b) } I_2 = \int_0^1 (2x+1)^3 dx$$

$$\text{c) } I_3 = \int_0^1 (1-4x)^3 dx$$

$$\text{d) } I_4 = \int_0^1 (x-1)(x^2-2x+5)^3 dx$$

$$\text{e) } I_5 = \int_0^1 (2x-3)(x^2-3x+1)^3 dx$$

Bài giải

$$\text{a) } I_1 = \int_0^1 x^3 dx = \frac{x^4}{4} \Big|_0^1 = \frac{1}{4}$$

$$\text{b) } I_2 = \int_0^1 (2x+1)^3 dx \quad \text{Chú ý: } d(2x+1) = 2dx \Rightarrow dx = \frac{1}{2}d(2x+1)$$

$$\Rightarrow I_2 = \int_0^1 (2x+1)^3 dx = \frac{1}{2} \int_0^1 (2x+1)^3 d(2x+1) = \frac{1}{2} \frac{(2x+1)^4}{4} \Big|_0^1 = \frac{81}{8} - \frac{1}{8} = 10$$

$$\text{c) } I_3 = \int_0^1 (1-4x)^3 dx \quad \text{Chú ý: } d(1-4x) = -4dx \Rightarrow dx = -\frac{1}{4}d(1-4x)$$

$$\Rightarrow I_3 = \int_0^1 (1-4x)^3 dx = -\frac{1}{4} \int_0^1 (1-4x)^3 d(1-4x) = -\frac{1}{4} \frac{(1-4x)^4}{4} \Big|_0^1 = -\frac{81}{16} + \frac{1}{16} = -5$$

$$\text{d) } I_4 = \int_0^1 (x-1)(x^2-2x+5)^3 dx \quad \text{Chú ý: } d(x^2-2x+5) = (2x-2)dx \Rightarrow (x-1)dx = \frac{1}{2}d(x^2-2x+5)$$

$$\Rightarrow I_4 = \int_0^1 (x-1)(x^2-2x+5)^3 dx = \frac{1}{2} \int_0^1 (x^2-2x+5)^3 d(x^2-2x+5)$$

$$= \frac{1}{2} \cdot \frac{(x^2-2x+5)^4}{4} \Big|_0^1 = 162 - \frac{615}{8} = \frac{671}{8}$$

$$\text{e) } I_5 = \int_0^1 (2x-3)(x^2-3x+1)^3 dx \quad \text{Chú ý: } d(x^2-3x+1) = (2x-3)dx$$

$$\Rightarrow I_5 = \int_0^1 (2x-3)(x^2-3x+1)^3 dx = \int_0^1 (x^2-3x+1)^3 d(x^2-3x+1)$$

$$= \frac{(x^2-3x+1)^4}{4} \Big|_0^1 = \frac{1}{4} - \frac{1}{4} = 0$$

HT 2. Tính các tích phân sau:

$$\text{a) } I_1 = \int_0^1 \sqrt{x} dx$$

$$\text{b) } I_2 = \int_2^7 \sqrt{x+2} dx$$

$$\text{c) } I_3 = \int_0^4 \sqrt{2x+1} dx$$

$$\text{d) } I_4 = \int_0^1 x\sqrt{1+x^2} dx$$

$$\text{e) } I_5 = \int_0^1 x\sqrt{1-x^2} dx$$

$$\text{f) } I_6 = \int_0^1 (1-x)\sqrt{x^2-2x+3} dx$$

$$\text{g) } I_7 = \int_0^1 x^2\sqrt{x^3+1} dx$$

$$\text{h) } I_8 = \int_0^1 (x^2-2x)\sqrt{x^3-3x^2+2} dx$$

Bài giải

$$\text{a) } I_1 = \int_0^1 \sqrt{x} dx = \frac{2}{3} x\sqrt{x} \Big|_0^1 = \frac{2}{3}$$

$$\text{b) } I_2 = \int_2^7 \sqrt{x+2} dx = \frac{2}{3} (x+2)\sqrt{x+2} \Big|_2^7 = 18 - \frac{16}{3} = \frac{38}{3}$$

$$\text{c) } I_3 = \int_0^4 \sqrt{2x+1} dx = \frac{1}{2} \int_0^4 \sqrt{2x+1} d(2x+1) = \frac{1}{2} \cdot \frac{2}{3} (2x+1)\sqrt{2x+1} \Big|_0^4 = 9 - \frac{1}{3} = \frac{26}{3}$$

$$\text{d) } I_4 = \int_0^1 x\sqrt{1+x^2} dx = \frac{1}{2} \int_0^1 \sqrt{1+x^2} d(1+x^2) = \frac{1}{2} \cdot \frac{2}{3} (1+x^2)\sqrt{1+x^2} \Big|_0^1 = \frac{2\sqrt{2}}{3} - \frac{1}{3}$$

$$\text{e) } I_5 = \int_0^1 x\sqrt{1-x^2} dx = -\frac{1}{2} \int_0^1 \sqrt{1-x^2} d(1-x^2) = -\frac{1}{2} \cdot \frac{2}{3} (1-x^2)\sqrt{1-x^2} \Big|_0^1 = 0 + \frac{1}{3} = \frac{1}{3}$$

$$\text{f) } I_6 = \int_0^1 (1-x)\sqrt{x^2-2x+3} dx = -\frac{1}{2} \int_0^1 \sqrt{x^2-2x+3} d(x^2-2x+3)$$

$$= -\frac{1}{2} \cdot \frac{2}{3} (x^2-2x+3)\sqrt{x^2-2x+3} \Big|_0^1 = -\frac{2\sqrt{2}}{3} + \sqrt{3}$$

$$\text{g) } I_7 = \int_0^1 x^2 \sqrt{x^3 + 1} dx = \frac{1}{3} \int_0^1 \sqrt{x^3 + 1} d(x^3 + 1) = \frac{1}{3} \cdot \frac{2}{3} (x^3 + 1) \sqrt{x^3 + 1} \Big|_0^1 = \frac{4\sqrt{2} - 2}{9}$$

$$\begin{aligned} \text{h) } I_8 &= \int_0^1 (x^2 - 2x) \sqrt{x^3 - 3x^2 + 2} dx = \frac{1}{3} \int_0^1 \sqrt{x^3 - 3x^2 + 2} d(x^3 - 3x^2 + 2) \\ &= \frac{1}{3} \cdot \frac{2}{3} (x^3 - 3x^2 + 2) \sqrt{x^3 - 3x^2 + 2} \Big|_0^1 = 0 - \frac{4\sqrt{2}}{9} = -\frac{4\sqrt{2}}{9} \end{aligned}$$

HT 3. Tính các tích phân sau:

$$\begin{array}{lll} \text{a) } I_1 = \int_1^4 \frac{dx}{\sqrt{x}} & \text{b) } I_2 = \int_0^1 \frac{dx}{\sqrt{2x+1}} & \text{c) } I_3 = \int_{-1}^0 \frac{dx}{\sqrt{1-2x}} \\ \text{d) } I_4 = \int_0^1 \frac{(x+1)dx}{\sqrt{x^2+2x+2}} & \text{e) } I_5 = \int_0^1 \frac{(x-2)dx}{\sqrt{x^2-4x+5}} \end{array}$$

Bài giải

$$\text{a) } I_1 = \int_1^4 \frac{dx}{\sqrt{x}} = 2\sqrt{x} \Big|_1^4 = 4 - 2 = 2$$

$$\text{b) } I_2 = \int_0^1 \frac{dx}{\sqrt{2x+1}} = \frac{1}{2} \int_0^1 \frac{d(2x+1)}{\sqrt{2x+1}} = \sqrt{2x+1} \Big|_0^1 = \sqrt{3} - 1$$

$$\text{c) } I_3 = \int_{-1}^0 \frac{dx}{\sqrt{1-2x}} = -\frac{1}{2} \int_{-1}^0 \frac{d(1-2x)}{\sqrt{1-2x}} = -\sqrt{1-2x} \Big|_{-1}^0 = -1 + \sqrt{3}$$

$$\text{d) } I_4 = \int_0^1 \frac{(x+1)dx}{\sqrt{x^2+2x+2}} = \frac{1}{2} \int_0^1 \frac{d(x^2+2x+2)}{\sqrt{x^2+2x+2}} = \sqrt{x^2+2x+2} \Big|_0^1 = \sqrt{5} - \sqrt{2}$$

$$\text{e) } I_5 = \int_0^1 \frac{(x-2)dx}{\sqrt{x^2-4x+5}} = \frac{1}{2} \int_0^1 \frac{d(x^2-4x+5)}{\sqrt{x^2-4x+5}} = \sqrt{x^2-4x+5} \Big|_0^1 = \sqrt{2} - \sqrt{5}$$

HT 4. Tính các tích phân sau:

$$\begin{array}{lll} \text{a) } I_1 = \int_1^e \frac{dx}{x} & \text{b) } I_2 = \int_{-1}^0 \frac{dx}{1-2x} & \text{c) } I_3 = \int_0^1 \frac{x dx}{x^2+1} \\ \text{d) } I_4 = \int_0^1 \frac{(x+1)dx}{x^2+2x+2} & \text{e) } I_5 = \int_0^1 \frac{x-2}{x^2-4x+5} dx \end{array}$$

Bài giải

$$\text{a) } I_1 = \int_1^e \frac{dx}{x} = \ln|x| \Big|_1^e = \ln e - \ln 1 = 1$$

$$\text{b) } I_2 = \int_{-1}^0 \frac{dx}{1-2x} = -\frac{1}{2} \int_{-1}^0 \frac{d(1-2x)}{1-2x} = -\frac{1}{2} \ln|1-2x| \Big|_{-1}^0 = -\frac{1}{2} (\ln 1 - \ln 3) = \frac{\ln 3}{2}$$

$$\text{c) } I_3 = \int_0^1 \frac{x dx}{x^2+1} = \frac{1}{2} \int_0^1 \frac{d(x^2+1)}{x^2+1} = \frac{1}{2} \ln|x^2+1| \Big|_0^1 = \frac{1}{2} (\ln 2 - \ln 1) = \frac{\ln 2}{2}$$

$$\text{d) } I_4 = \int_0^1 \frac{(x+1)dx}{x^2+2x+2} = \frac{1}{2} \int_0^1 \frac{d(x^2+2x+2)}{x^2+2x+2} = \frac{1}{2} \ln|x^2+2x+2| \Big|_0^1 = \frac{1}{2} (\ln 5 - \ln 2) = \frac{1}{2} \ln \frac{5}{2}$$

$$\text{e) } I_5 = \int_0^1 \frac{x-2}{x^2-4x+5} dx = \frac{1}{2} \int_0^1 \frac{d(x^2-4x+5)}{x^2-4x+5} = \frac{1}{2} \ln|x^2-4x+5| \Big|_0^1 = \frac{1}{2} (\ln 2 - \ln 5) = \frac{1}{2} \ln \frac{2}{5}$$

HT 5. Tính các tích phân sau:

$$\text{a) } I_1 = \int_1^2 \frac{dx}{x^2}$$

$$\text{b) } I_2 = \int_{-1}^0 \frac{dx}{(2x-1)^2}$$

$$\text{c) } I_3 = \int_0^1 \frac{dx}{(3x+1)^2}$$

Bài giải

$$\text{a) } I_1 = \int_1^2 \frac{dx}{x^2} = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\text{b) } I_2 = \int_{-1}^0 \frac{dx}{(2x-1)^2} = \frac{1}{2} \int_{-1}^0 \frac{d(2x-1)}{(2x-1)^2} = -\frac{1}{2} \cdot \frac{1}{2x-1} \Big|_{-1}^0 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

$$\text{c) } I_3 = \int_0^1 \frac{dx}{(3x+1)^2} = \frac{1}{3} \int_0^1 \frac{d(3x+1)}{(3x+1)^2} = -\frac{1}{3} \cdot \frac{1}{3x+1} \Big|_0^1 = -\frac{1}{12} + \frac{1}{4} = \frac{1}{6}$$

HT 6. Tính các tích phân sau:

$$\text{a) } I_1 = \int_0^1 e^{3x} dx$$

$$\text{b) } I_2 = \int_0^1 e^x (2e^x + 1)^3 dx$$

$$\text{c) } I_3 = \int_0^1 e^x (1 - 4e^x)^3 dx$$

$$\text{d) } I_4 = \int_0^1 \frac{e^x dx}{e^x + 1}$$

$$\text{e) } I_5 = \int_1^2 \frac{e^{2x} dx}{(e^{2x} - 1)^2}$$

$$\text{f) } I_6 = \int_1^2 \frac{e^{2x} dx}{(1 - 3e^{2x})^3}$$

$$\text{g) } I_7 = \int_0^1 e^x \sqrt{2e^x + 1} dx$$

$$\text{h) } I_8 = \int_0^1 e^{2x} \sqrt{1 + 3e^{2x}} dx$$

$$\text{i) } I_9 = \int_0^1 \frac{e^x dx}{\sqrt{e^x + 1}}$$

$$\text{a) } I_1 = \int_0^1 e^{3x} dx = \frac{1}{3} e^{3x} \Big|_0^1 = \frac{e^3}{3} - \frac{1}{3}$$

$$\begin{aligned} \text{b) } I_2 &= \int_0^1 e^x (2e^x + 1)^3 dx = \frac{1}{2} \int_0^1 (2e^x + 1)^3 d(2e^x + 1) = \frac{1}{2} \cdot \frac{(2e^x + 1)^4}{4} \Big|_0^1 \\ &= \frac{1}{2} \left(\frac{(2e + 1)^4}{4} - \frac{81}{4} \right) = \frac{(2e + 1)^4}{8} - \frac{81}{8} \end{aligned}$$

$$\begin{aligned} \text{c) } I_3 &= \int_0^1 e^x (1 - 4e^x)^3 dx = -\frac{1}{4} \int_0^1 (1 - 4e^x)^3 d(1 - 4e^x) \\ &= -\frac{1}{4} \cdot \frac{(1 - 4e^x)^4}{4} \Big|_0^1 = -\frac{1}{4} \left(\frac{(1 - 4e)^4}{4} - \frac{81}{4} \right) = \frac{81 - (1 - 4e)^4}{16} \end{aligned}$$

$$\text{d) } I_4 = \int_0^1 \frac{e^x dx}{e^x + 1} = \int_0^1 \frac{d(e^x + 1)}{e^x + 1} = \ln |e^x + 1| \Big|_0^1 = \ln(e + 1) - \ln 2 = \ln \frac{e + 1}{2}$$

$$\text{e) } I_5 = \int_1^2 \frac{e^{2x} dx}{(e^{2x} - 1)^2} = \frac{1}{2} \int_1^2 \frac{d(e^{2x} - 1)}{(e^{2x} - 1)^2} = -\frac{1}{2} \cdot \frac{1}{e^{2x} - 1} \Big|_1^2 = -\frac{1}{2(e^4 - 1)} + \frac{1}{2(e^2 - 1)} = \frac{e^2}{2(e^4 - 1)}$$

$$\text{f) } I_6 = \int_1^2 \frac{e^{2x} dx}{(1 - 3e^{2x})^3} = -\frac{1}{6} \int_1^2 \frac{d(1 - 3e^{2x})}{(1 - 3e^{2x})^3} = -\frac{1}{6} \cdot \frac{-1}{2(1 - 3e^{2x})^2} \Big|_1^2 = \frac{1}{12(1 - 3e^4)} - \frac{1}{12(1 - 3e^2)}$$

$$\text{g) } I_7 = \int_0^1 e^x \sqrt{2e^x + 1} dx = \frac{1}{2} \int_0^1 \sqrt{2e^x + 1} d(2e^x + 1) = \frac{1}{2} \cdot \frac{2}{3} (2e^x + 1) \sqrt{2e^x + 1} \Big|_0^1 = \frac{1}{3} (2e + 1) \sqrt{2e + 1} - \sqrt{3}$$

$$\text{h) } I_8 = \int_0^1 e^{2x} \sqrt{1 + 3e^{2x}} dx = \frac{1}{6} \int_0^1 \sqrt{1 + 3e^{2x}} d(1 + 3e^{2x}) = \frac{1}{6} \cdot \frac{2}{3} (1 + 3e^{2x}) \sqrt{1 + 3e^{2x}} \Big|_0^1 = \frac{1}{9} (1 + 3e^2) \sqrt{1 + 3e^2} - \frac{8}{9}$$

$$\text{i) } I_9 = \int_0^1 \frac{e^x dx}{\sqrt{e^x + 1}} = \int_0^1 \frac{d(e^x + 1)}{\sqrt{e^x + 1}} = 2\sqrt{e^x + 1} \Big|_0^1 = 2\sqrt{e + 1} - 2$$

HT 7. Tính các tích phân sau:

$$\text{a) } I_1 = \int_1^e \frac{\ln x}{x} dx$$

$$\text{b) } I_2 = \int_1^e \frac{3 \ln x + 1}{x} dx$$

$$\text{c) } I_3 = \int_1^e \frac{(3 \ln x + 1)^3}{x} dx$$

$$\text{d) } I_4 = \int_1^e \frac{4 \ln^3 x + 3 \ln^2 x - 2 \ln x + 1}{x} dx$$

$$\text{e) } I_5 = \int_e^{e^2} \frac{dx}{x \ln x}$$

$$\text{f) } I_6 = \int_1^e \frac{dx}{x(3 \ln x + 1)}$$

$$\text{g) } I_7 = \int_1^e \frac{\sqrt{3 \ln x + 1} dx}{x}$$

$$\text{h) } I_8 = \int_1^e \frac{dx}{x\sqrt{3 \ln x + 1}}$$

Bài giải

$$\text{a) } I_1 = \int_1^e \frac{\ln x}{x} dx = \int_1^e \ln x d(\ln x) = \frac{\ln^2 x}{2} \Big|_1^e = \frac{\ln^2 e}{2} - \frac{\ln^2 1}{2} = \frac{1}{2}$$

$$\text{b) } I_2 = \int_1^e \frac{3 \ln x + 1}{x} dx = \int_1^e (3 \ln x + 1) d(\ln x) = \left(\frac{3 \ln^2 x}{2} + \ln x \right) \Big|_1^e = \left(\frac{3}{2} + 1 \right) - 0 = \frac{5}{2}$$

$$\text{c) } I_3 = \int_1^e \frac{(3 \ln x + 1)^3}{x} dx = \frac{1}{3} \int_1^e (3 \ln x + 1)^3 d(3 \ln x + 1) = \frac{1}{3} \cdot \frac{(3 \ln x + 1)^4}{4} \Big|_1^e = \frac{64}{3} - \frac{1}{12} = \frac{85}{4}$$

$$\text{d) } I_4 = \int_1^e \frac{4 \ln^3 x + 3 \ln^2 x - 2 \ln x + 1}{x} dx = \int_1^e (4 \ln^3 x + 3 \ln^2 x - 2 \ln x + 1) d(\ln x)$$

$$= (\ln^4 x + \ln^3 x - \ln^2 x + \ln x) \Big|_1^e = (1 + 1 - 1 + 1) - 0 = 2$$

$$\text{e) } I_5 = \int_e^{e^2} \frac{dx}{x \ln x} = \int_e^{e^2} \frac{d(\ln x)}{\ln x} = \ln(\ln x) \Big|_e^{e^2} = \ln(\ln e^2) - \ln(\ln e) = \ln 2$$

$$\text{f) } I_6 = \int_1^e \frac{dx}{x(3 \ln x + 1)} = \frac{1}{3} \int_1^e \frac{d(3 \ln x + 1)}{3 \ln x + 1} = \frac{1}{3} \ln(3 \ln x + 1) \Big|_1^e = \frac{1}{3} (\ln 4 - \ln 1) = \frac{\ln 4}{3}$$

$$\text{g) } I_7 = \int_1^e \frac{\sqrt{3 \ln x + 1} dx}{x} = \frac{1}{3} \int_1^e \sqrt{3 \ln x + 1} d(3 \ln x + 1) = \frac{1}{3} \cdot \frac{2}{3} (3 \ln x + 1) \sqrt{3 \ln x + 1} \Big|_1^e = \frac{16}{9} - \frac{2}{9} = \frac{14}{9}$$

$$\text{h) } I_8 = \int_1^e \frac{dx}{x\sqrt{3 \ln x + 1}} = \frac{1}{3} \int_1^e \frac{d(3 \ln x + 1)}{\sqrt{3 \ln x + 1}} = \frac{1}{3} \cdot 2 \sqrt{3 \ln x + 1} \Big|_1^e = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

HT 8. Tính các tích phân sau:

$$\text{a) } I_1 = \int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx$$

$$\text{b) } I_2 = \int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$$

$$\text{c) } I_3 = \int_0^{\frac{\pi}{4}} \sin^3 2x \cos 2x dx$$

$$\text{d) } I_4 = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx$$

$$\text{e) } I_5 = \int_0^{\frac{\pi}{2}} \sin x \sqrt{3 \cos x + 1} dx$$

$$\text{f) } I_6 = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{3 \sin x + 1}} dx$$

Giải

$$\text{a) } I_1 = \int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = - \int_0^{\frac{\pi}{2}} \cos^2 x d(\cos x) = - \frac{\cos^3 x}{3} \Big|_0^{\frac{\pi}{2}} = \frac{1}{3}$$

$$\text{b) } I_2 = \int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx = \int_0^{\frac{\pi}{2}} \sin^2 x d(\sin x) = \frac{\sin^3 x}{3} \Big|_0^{\frac{\pi}{2}} = \frac{1}{3}$$

$$\text{c) } I_3 = \int_0^{\frac{\pi}{4}} \sin^3 2x \cos 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin^3 2x d(\sin 2x) = \frac{\sin^4 2x}{8} \Big|_0^{\frac{\pi}{4}} = \frac{1}{8}$$

$$\text{d) } I_4 = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx = - \int_0^{\frac{\pi}{4}} \frac{d(\cos x)}{\cos x} = - \ln(\cos x) \Big|_0^{\frac{\pi}{4}} = - \ln \frac{\sqrt{2}}{2} + \ln 1 = - \ln \frac{\sqrt{2}}{2}$$

$$\text{e) } I_5 = \int_0^{\frac{\pi}{2}} \sin x \sqrt{3 \cos x + 1} dx = \frac{1}{3} \int_0^{\frac{\pi}{2}} \sqrt{3 \cos x + 1} d(3 \cos x + 1) = \frac{1}{2} \cdot \frac{2}{3} (3 \cos x + 1) \sqrt{3 \cos x + 1} \Big|_0^{\frac{\pi}{2}} = \frac{1}{3} - \frac{4}{3} = -1$$

$$\text{f) } I_6 = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{3 \sin x + 1}} dx = \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{d(3 \sin x + 1)}{\sqrt{3 \sin x + 1}} = \frac{2}{3} \sqrt{3 \sin x + 1} \Big|_0^{\frac{\pi}{2}} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

<http://www.Luuhuythuong.blogspot.com>

PHẦN II TÍCH PHẦN HÀM HỮU TỶ

<http://www.LuuHuyThuong.blogspot.com>

I. DẠNG 1: $\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$

HT 1. Tính các tích phân sau:

a) $\int_0^1 \frac{dx}{3x+1}$

b) $\int_{-1}^0 \frac{dx}{1-3x}$

c) $\int_0^1 \left(\frac{1}{2x+1} - \frac{3}{4-2x} \right) dx$

Giải

a) $\int_0^1 \frac{dx}{3x+1} = \frac{1}{3} \ln|3x+1| \Big|_0^1 = \frac{1}{3} (\ln 4 - \ln 1) = \frac{\ln 4}{3}$

b) $\int_{-1}^0 \frac{dx}{1-3x} = -\frac{1}{3} \ln|1-3x| \Big|_{-1}^0 = -\frac{1}{3} (\ln 1 - \ln 4) = -\frac{\ln 4}{3}$

c) $\int_0^1 \left(\frac{1}{2x+1} - \frac{3}{4-2x} \right) dx = \left(\frac{1}{2} \ln|2x+1| + \frac{3}{2} \ln|4-2x| \right) \Big|_0^1 = \left(\frac{1}{2} \ln 3 + \frac{3}{2} \ln 2 \right) - \left(\frac{1}{2} \ln 1 + \frac{3}{2} \ln 4 \right)$
 $= \frac{1}{2} \ln 3 + \frac{3}{2} \ln \frac{1}{2}$

HT 2. Tính các tích phân sau:

a) $I_1 = \int_1^2 \frac{x^4 + 3x^3 - 2x^2 + 5x - 1}{x^2} dx$ b) $I_2 = \int_0^1 \frac{x^3 - 3x^2 + 2x - 1}{x-2} dx$ c) $I_3 = \int_{-1}^0 \frac{2x^3 - 3x^2 + 4x - 1}{1-2x} dx$

Giải

a) $I_1 = \int_1^2 \frac{x^4 + 3x^3 - 2x^2 + 5x - 1}{x^2} dx = \int_1^2 \left(x^2 + 3x - 2 + \frac{5}{x} - \frac{1}{x^2} \right) dx$
 $= \left(\frac{x^3}{3} + \frac{3x^2}{2} - 2x + 5 \ln x + \frac{1}{x} \right) \Big|_1^2 = \left(\frac{8}{3} + 6 - 4 + 5 \ln 2 + \frac{1}{2} \right) - \left(\frac{1}{3} + \frac{3}{2} - 2 + 5 \ln 1 + 1 \right) = \frac{13}{3} + 5 \ln 2$

b) $I_2 = \int_0^1 \frac{x^3 - 3x^2 + 2x - 1}{x-2} dx = \int_0^1 \left(x^2 - x - \frac{1}{x-2} \right) dx$
 $= \left(\frac{x^3}{3} - \frac{x^2}{2} - \ln|x-2| \right) \Big|_0^1 = \left(\frac{1}{3} - \frac{1}{2} - \ln 1 \right) - (-\ln 2) = \ln 2 - \frac{1}{6}$

c) $I_3 = \int_{-1}^0 \frac{2x^3 - 3x^2 + 4x - 1}{1-2x} dx = \int_{-1}^0 \left(-x^2 + x - \frac{3}{2} + \frac{1}{2(-2x+1)} \right) dx$

$$= \left(-\frac{x^3}{3} + \frac{x^2}{2} - \frac{3}{2}x - \frac{1}{4}\ln|-2x+1| \right) \Big|_{-1}^0$$

$$= \left(-\frac{1}{4}\ln 1 \right) - \left(\frac{1}{3} + \frac{1}{2} + \frac{3}{2} - \frac{1}{4}\ln 3 \right) = \frac{\ln 3}{4} - \frac{7}{3}$$

II. DẠNG 2: $\int \frac{dx}{ax^2 + bx + c}$

HT 3. Tính các tích phân sau (mẫu số có hai nghiệm phân biệt)

a) $\int_0^1 \frac{dx}{(x+1)(x+2)}$

b) $\int_0^1 \frac{dx}{(x+1)(3-x)}$

c) $\int_0^1 \frac{dx}{(x+1)(2x+3)}$

Giải

a) $\int_0^1 \frac{dx}{(x+1)(x+2)} = \int_0^1 \frac{(x+2) - (x+1)}{(x+1)(x+2)} dx = \int_0^1 \left(\frac{1}{x+1} - \frac{1}{x+2} \right) dx$

$$= \left(\ln|x+1| - \ln|x+2| \right) \Big|_0^1 = \ln \left| \frac{x+1}{x+2} \right| \Big|_0^1 = \ln \frac{2}{3} - \ln \frac{1}{2} = \ln \frac{4}{3}$$

b) $\int_0^1 \frac{dx}{(x+1)(3-x)} = \frac{1}{4} \int_0^1 \frac{(x+1) + (3-x)}{(x+1)(3-x)} dx = \frac{1}{4} \int_0^1 \left(\frac{1}{3-x} + \frac{1}{x+1} \right) dx$

$$= \frac{1}{4} \left(-\ln|3-x| + \ln|x+1| \right) \Big|_0^1 = \frac{1}{4} \ln \left| \frac{x+1}{3-x} \right| \Big|_0^1 = \frac{1}{4} \left(\ln 1 - \ln \frac{1}{3} \right) = -\frac{\ln 3}{4}$$

c) $\int_0^1 \frac{dx}{(x+1)(2x+3)} = \int_0^1 \frac{(2x+3) - 2(x+1)}{(x+1)(2x+3)} dx = \int_0^1 \left(\frac{1}{x+1} - \frac{2}{2x+3} \right) dx$

$$= \left(\ln|x+1| - \ln|2x+3| \right) \Big|_0^1 = \ln \left| \frac{x+1}{2x+3} \right| \Big|_0^1 = \ln \frac{2}{5} - \ln \frac{1}{3} = \ln \frac{6}{5}$$

HT 4. Tính các tích phân sau:

a) $\int_0^1 \frac{dx}{x^2 - x - 12}$

b) $\int_{-1}^0 \frac{dx}{2x^2 - 5x + 2}$

c) $\int_1^2 \frac{dx}{1 - 2x - 3x^2}$

Giải

a) $\int_0^1 \frac{dx}{x^2 - x - 12} = \int_0^1 \frac{dx}{(x+3)(x-4)} = \frac{1}{7} \int_0^1 \frac{(x+3) - (x-4)}{(x+3)(x-4)} dx$

$$= \frac{1}{7} \int_0^1 \left(\frac{1}{x-4} - \frac{1}{x+3} \right) dx = \frac{1}{7} \left(\ln|x-4| - \ln|x+3| \right) \Big|_0^1 = \frac{1}{7} \ln \left| \frac{x-4}{x+3} \right| \Big|_0^1$$

$$= \frac{1}{7} \left(\ln \frac{3}{4} - \ln \frac{4}{3} \right) = \frac{1}{7} \ln \frac{9}{16}$$

b) $\int_{-1}^0 \frac{dx}{2x^2 - 5x + 2} = \int_{-1}^0 \frac{dx}{-1} \frac{dx}{2(x-2)(x-\frac{1}{2})} = \int_{-1}^0 \frac{dx}{(x-2)(2x-1)} = \frac{1}{3} \int_{-1}^0 \frac{(2x-1) - 2(x-2)}{(x-2)(2x-1)} dx$

$$= \frac{1}{3} \int_{-1}^0 \left(\frac{1}{x-2} - \frac{2}{2x-1} \right) dx = \frac{1}{3} \left(\ln|x-2| - \ln|2x-1| \right) \Big|_{-1}^0$$

$$= \frac{1}{3} \ln \left| \frac{x-2}{2x-1} \right| \Big|_{-1}^0 = \frac{1}{3} (\ln 2 - \ln 1) = \frac{\ln 2}{3}$$

$$\text{c) } \int_1^2 \frac{dx}{1-2x-3x^2} = \int_1^2 \frac{dx}{-3(x+1)(x-\frac{1}{3})} = \int_1^2 \frac{dx}{(x+1)(1-3x)} = \frac{1}{4} \int_1^2 \frac{3(x+1) + (1-3x)}{(x+1)(1-3x)} dx$$

$$= \frac{1}{4} \int_1^2 \left(\frac{3}{1-3x} + \frac{1}{x+1} \right) dx = \frac{1}{4} \left(-\ln|1-3x| + \ln|x+1| \right) \Big|_1^2 = \frac{1}{4} \ln \left| \frac{x+1}{1-3x} \right| \Big|_1^2 = \frac{1}{4} (\ln \frac{3}{5} - \ln 1) = \frac{1}{4} \ln \frac{3}{5}$$

HT 5. Tính các tích phân sau: (Mẫu số có nghiệm kép)

$$\text{a) } \int_1^2 \frac{dx}{x^2}$$

$$\text{b) } \int_0^1 \frac{dx}{(3x+1)^2}$$

$$\text{c) } \int_{-1}^0 \frac{dx}{(1-2x)^2}$$

$$\text{d) } \int_{-1}^0 \frac{dx}{9x^2-6x+1}$$

$$\text{e) } \int_{-1}^0 \frac{dx}{-16x^2+8x-1}$$

Giải

$$\text{a) } \int_1^2 \frac{dx}{x^2} = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\text{b) } \int_0^1 \frac{dx}{(3x+1)^2} = -\frac{1}{3} \cdot \frac{1}{3x+1} \Big|_0^1 = -\left(\frac{1}{12} - \frac{1}{3} \right) = \frac{1}{4}$$

$$\text{c) } \int_{-1}^0 \frac{dx}{(1-2x)^2} = \int_{-1}^0 \frac{dx}{(2x-1)^2} = -\frac{1}{2} \cdot \frac{1}{2x-1} \Big|_{-1}^0 = -\left(-\frac{1}{2} + \frac{1}{6} \right) = \frac{1}{3}$$

$$\text{d) } \int_{-1}^0 \frac{dx}{9x^2-6x+1} = \int_{-1}^0 \frac{dx}{(3x-1)^2} = -\frac{1}{3} \cdot \frac{1}{3x-1} \Big|_{-1}^0 = -\left(-\frac{1}{3} + \frac{1}{12} \right) = \frac{1}{4}$$

$$\text{e) } \int_{-1}^0 \frac{dx}{-16x^2+8x-1} = -\int_{-1}^0 \frac{dx}{16x^2-8x+1} = -\int_{-1}^0 \frac{dx}{(4x-1)^2} = \frac{1}{4} \cdot \frac{1}{4x-1} \Big|_{-1}^0 = -\frac{1}{4} + \frac{1}{20} = -\frac{1}{5}$$

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HT 6. Tính các tích phân sau: (Mẫu số vô nghiệm)

$$\text{a) } I_1 = \int_0^1 \frac{dx}{x^2+1}$$

$$\text{b) } \int_0^{\sqrt{3}} \frac{dx}{x^2+3}$$

$$\text{c) } \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{2x^2+3}$$

Giải

$$\text{a) } I_1 = \int_0^1 \frac{dx}{x^2+1}$$

$$\text{Đặt: } x = \tan t \quad \left(t \in \left(-\frac{\pi}{2}; \frac{\pi}{2} \right) \right)$$

$$\Rightarrow dx = \frac{dt}{\cos^2 t}$$

$$\text{Đổi cận: Với } x = 0 \Rightarrow t = 0$$

$$\text{Với } x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t (\tan^2 t + 1)} = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} = \int_0^{\frac{\pi}{4}} dt = t \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

$$\text{b) } I_2 = \int_0^{\sqrt{3}} \frac{dx}{x^2 + 3}$$

$$\text{Đặt: } x = \sqrt{3} \tan t \text{ Với } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$$

$$\Rightarrow dx = \frac{\sqrt{3} dt}{\cos^2 t}$$

$$\text{Đổi cận: Với } x = 0 \Rightarrow t = 0; \text{ Với } x = \sqrt{3} \Rightarrow t = \frac{\pi}{4}$$

$$\Rightarrow I_2 = \int_0^{\frac{\pi}{4}} \frac{\sqrt{3} dt}{\cos^2 t (3 \tan^2 t + 3)} = \frac{\sqrt{3}}{3} \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} = \frac{\sqrt{3}}{3} \int_0^{\frac{\pi}{4}} dt = \frac{\sqrt{3}}{3} t \Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{3}\pi}{12}$$

$$\text{c) } I_3 = \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{2x^2 + 3} = \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{2 \left(x^2 + \frac{3}{2}\right)} = \frac{1}{2} \int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{x^2 + \frac{3}{2}}$$

$$\text{Đặt: } x = \sqrt{\frac{3}{2}} \tan t \text{ Với } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$$

$$\Rightarrow dx = \frac{\sqrt{6}}{2} \frac{dt}{\cos^2 t}$$

$$\text{Đổi cận: Với } x = 0 \Rightarrow t = 0; \text{ Với } x = \frac{\sqrt{2}}{2} \Rightarrow t = \frac{\pi}{6}$$

$$\Rightarrow I_3 = \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{\sqrt{6} dt}{2 \cos^2 t \left(\frac{3}{2} \tan^2 t + \frac{3}{2}\right)} = \frac{\sqrt{6}}{6} \int_0^{\frac{\pi}{6}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} = \frac{\sqrt{6}}{6} \int_0^{\frac{\pi}{6}} dt = \frac{\sqrt{6}}{6} t \Big|_0^{\frac{\pi}{6}} = \frac{\sqrt{6}\pi}{36}$$

HT 7. Tính các tích phân sau: (Mẫu số vô nghiệm)

$$\text{a) } I_1 = \int_{-1}^0 \frac{dx}{(x+1)^2 + 1}$$

$$\text{b) } I_2 = \int_2^4 \frac{dx}{x^2 - 4x + 8}$$

$$\text{c) } I_3 = \int_0^1 \frac{dx}{x^2 + x + 1}$$

Giải

$$\text{a) } I_1 = \int_{-1}^0 \frac{dx}{(x+1)^2 + 1}$$

$$\text{Đặt: } x + 1 = \tan t \text{ Với } t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$$

$$\Rightarrow dx = \frac{dt}{\cos^2 t}$$

Đổi cận: Với $x = -1 \Rightarrow t = 0$; Với $x = 0 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t (\tan^2 t + 1)} = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} = \int_0^{\frac{\pi}{4}} dt = t \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

$$\text{b) } I_2 = \int_2^4 \frac{dx}{x^2 - 4x + 8} = \int_2^4 \frac{dx}{(x-2)^2 + 4}$$

Đặt: $x - 2 = 2 \tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\Rightarrow dx = \frac{2dt}{\cos^2 t}$$

Đổi cận: Với $x = 2 \Rightarrow t = 0$; Với $x = 4 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow I_2 = \int_0^{\frac{\pi}{4}} \frac{2dt}{\cos^2 t (4 \tan^2 t + 4)} = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} = \frac{1}{2} \int_0^{\frac{\pi}{4}} dt = \frac{1}{2} t \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{8}$$

$$\text{c) } I_3 = \int_0^1 \frac{dx}{x^2 + x + 1} = \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

Đặt: $x + \frac{1}{2} = \frac{\sqrt{3}}{2} \tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\Rightarrow dx = \frac{\sqrt{3}}{2} \cdot \frac{dt}{\cos^2 t}$$

Đổi cận: Với $x = 0 \Rightarrow t = \frac{\pi}{6}$; Với $x = 1 \Rightarrow t = \frac{\pi}{3}$

$$\Rightarrow I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{3} dt}{2 \cos^2 t \left(\frac{3}{4} \tan^2 t + \frac{3}{4}\right)} = \frac{2\sqrt{3}}{3} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} =$$

$$\frac{2\sqrt{3}}{3} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dt = \frac{2\sqrt{3}}{3} t \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{2\sqrt{3}\pi}{9} - \frac{2\sqrt{3}\pi}{18} = \frac{2\sqrt{3}\pi}{18}$$

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III. Dạng 3: $\int \frac{mx+n}{ax^2+bx+c} dx$

HT 8. Tính các tích phân sau: (Mẫu số có 2 nghiệm phân biệt)

a) $I_1 = \int_0^1 \frac{x-1}{x^2+4x+3} dx$

b) $I_2 = \int_{-1}^0 \frac{2x+10}{-x^2+x+2} dx$

c) $I_3 = \int_{-1}^0 \frac{7-4x}{-2x^2-3x+2} dx$

Giải

a) $I_1 = \int_0^1 \frac{x-1}{x^2+4x+3} dx = \int_0^1 \frac{(x-1)dx}{(x+1)(x+3)}$

Xét đồng nhất thức: $\frac{x-1}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} = \frac{Ax+A+Bx+3B}{(x+3)(x+1)} = \frac{(A+B)x+A+3B}{(x+3)(x+1)}$

Đồng nhất thức hai vế ta được: $\begin{cases} A+B=1 \\ A+3B=-1 \end{cases} \Leftrightarrow \begin{cases} A=2 \\ B=-1 \end{cases}$

Vậy, $I_1 = \int_0^1 \left(\frac{2}{x+3} - \frac{1}{x+1} \right) dx = (2\ln|x+3| - \ln|x+1|) \Big|_0^1$

$= (2\ln 4 - \ln 2) - (2\ln 3 - \ln 1) = 2\ln \frac{4}{3} - \ln 2$

b) $\int_{-1}^0 \frac{2x+10}{-x^2+x+2} dx = \int_{-1}^0 \frac{2x+10}{(x+2)(1-x)} dx$

Xét đồng nhất thức: $\frac{2x+10}{(x+2)(1-x)} = \frac{A}{x+2} + \frac{B}{1-x} = \frac{A-Ax+Bx+2B}{(x+2)(1-x)} = \frac{(B-A)x+A+2B}{(x+2)(1-x)}$

Đồng nhất thức hai vế ta được: $\begin{cases} B-A=2 \\ A+2B=10 \end{cases} \Leftrightarrow \begin{cases} A=2 \\ B=4 \end{cases}$

Vậy, $I_2 = \int_{-1}^0 \left(\frac{2}{x+2} + \frac{4}{1-x} \right) dx = (2\ln|x+2| - 4\ln|1-x|) \Big|_{-1}^0$

$= (2\ln 2 - 4\ln 1) - (2\ln 1 - 4\ln 2) = 2\ln 2 + 4\ln 2 = \ln 4 + \ln 16 = \ln 64$

c) $I_3 = \int_{-1}^0 \frac{7-4x}{-2x^2-3x+2} dx = \int_{-1}^0 \frac{7-4x}{(x+2)(1-2x)} dx$

Xét đồng nhất thức: $\frac{7-4x}{(x+2)(1-2x)} = \frac{A}{x+2} + \frac{B}{1-2x} = \frac{A-2Ax+Bx+2B}{(x+2)(1-2x)} = \frac{(B-2A)x+A+2B}{(x+2)(1-2x)}$

Đồng nhất thức hai vế ta được: $\begin{cases} B-2A=-4 \\ A+2B=7 \end{cases} \Leftrightarrow \begin{cases} A=3 \\ B=2 \end{cases}$

Vậy, $I_3 = \int_{-1}^0 \left(\frac{2}{1-2x} + \frac{3}{x+2} \right) dx = (-\ln|1-2x| + 3\ln|x+2|) \Big|_{-1}^0$

$= (-\ln 1 + 2\ln 2) - (-\ln 3 + 3\ln 2) = \ln 3 - \ln 2 = \ln \frac{3}{2}$

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HT 9. Tính các tích phân sau: (Mẫu số có nghiệm kép)

a) $I_1 = \int_0^1 \frac{(3x+1)dx}{x^2+2x+1}$

b) $I_2 = \int_{-1}^0 \frac{3x-1}{4x^2-4x+1} dx$

c) $I_3 = \int_0^1 \frac{3x+2}{4x^2+12x+9} dx$

Giải

$$\begin{aligned} \text{a) } I_1 &= \int_0^1 \frac{(3x+1)dx}{x^2+2x+1} = \int_0^1 \frac{3x+1}{(x+1)^2} dx = \int_0^1 \frac{3(x+1)-2}{(x+1)^2} dx = \int_0^1 \left(\frac{3}{x+1} - \frac{2}{(x+1)^2} \right) dx \\ &= \left(3 \ln|x+1| + \frac{2}{x+1} \right) \Big|_0^1 = (3 \ln 2 + 1) - (3 \ln 1 + 2) = 3 \ln 2 - 1 \end{aligned}$$

$$\begin{aligned} \text{b) } I_2 &= \int_{-1}^0 \frac{3x-1}{4x^2-4x+1} dx = \int_{-1}^0 \frac{3x-1}{(2x-1)^2} dx = \int_{-1}^0 \frac{\frac{3}{2}(2x-1) + \frac{1}{2}}{(2x-1)^2} dx \\ &= \int_{-1}^0 \left(\frac{3}{2} \cdot \frac{1}{2x-1} + \frac{1}{2} \cdot \frac{1}{(2x-1)^2} \right) dx = \left(\frac{3}{4} \ln|2x-1| - \frac{1}{4} \cdot \frac{1}{2x-1} \right) \Big|_{-1}^0 \\ &= \left(\frac{3}{4} \ln 1 + \frac{1}{4} \right) - \left(\frac{3}{4} \ln 3 + \frac{1}{12} \right) = -\frac{3}{4} \ln 3 + \frac{1}{6} \end{aligned}$$

$$\begin{aligned} \text{c) } I_3 &= \int_0^1 \frac{3x+2}{4x^2+12x+9} dx = \int_0^1 \frac{3x+2}{(2x+3)^2} dx = \int_0^1 \frac{\frac{3}{2}(2x+3) - \frac{5}{2}}{(2x+3)^2} dx \\ &= \int_0^1 \left(\frac{3}{2} \cdot \frac{1}{2x+3} - \frac{5}{2} \cdot \frac{1}{(2x+3)^2} \right) dx = \left(\frac{3}{4} \ln|2x+3| + \frac{5}{4} \cdot \frac{1}{2x+3} \right) \Big|_0^1 \\ &= \left(\frac{3}{4} \ln 5 + \frac{1}{4} \right) - \left(\frac{3}{4} \ln 3 + \frac{5}{12} \right) = \frac{3}{4} \ln \frac{5}{3} - \frac{1}{6} \end{aligned}$$

HT 10. Tính các tích phân sau: (Mẫu số vô nghiệm)

a) $I_1 = \int_0^1 \frac{3x+1}{x^2+1} dx$

b) $I_2 = \int_1^3 \frac{3x+2}{x^2-4x+5} dx$

c) $I_3 = \int_0^1 \frac{3x-1}{4x^2-4x+2} dx$

Giải

$$\text{a) } I_1 = \int_0^1 \frac{3x+1}{x^2+1} dx$$

Chú ý: $(x^2+1)' = 2x$ Nên: $I_1 = \int_0^1 \frac{\frac{3}{2} \cdot 2x + 1}{x^2+1} dx = \int_0^1 \left(\frac{3}{2} \cdot \frac{2x}{x^2+1} + \frac{1}{x^2+1} \right) dx = \frac{3}{2} \int_0^1 \frac{2x}{x^2+1} dx + \int_0^1 \frac{dx}{x^2+1}$

Xét: $M = \frac{3}{2} \int_0^1 \frac{2x}{x^2+1} dx = \frac{3}{2} \int_0^1 \frac{d(x^2+1)}{x^2+1} = \frac{3}{2} \ln|x^2+1| \Big|_0^1 = \frac{3}{2} (\ln 2 - \ln 1) = \frac{3 \ln 2}{2}$

Xét: $N = \int_0^1 \frac{dx}{x^2+1}$

$$\text{Đặt: } x = \tan t \quad \left(t \in \left(-\frac{\pi}{2}; \frac{\pi}{2} \right) \right)$$

$$\Rightarrow dx = \frac{dt}{\cos^2 t}$$

$$\text{Đổi cận: Với } x = 0 \Rightarrow t = 0$$

$$\text{Với } x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\Rightarrow M = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t (\tan^2 t + 1)} = \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t \cdot \frac{1}{\cos^2 t}} = \int_0^{\frac{\pi}{4}} dt = t \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

$$\text{Vậy, } I_1 = M + N = \frac{3 \ln 2}{2} + \frac{\pi}{4}$$

$$\text{b) } I_2 = \int_1^3 \frac{3x+2}{x^2-4x+5} dx$$

$$\text{Chú ý: } (x^2 - 4x + 5)' = 2x - 4$$

$$\text{Khi đó: } I_2 = \int_1^3 \frac{\frac{3}{2}(2x-4) + 8}{x^2 - 4x + 5} dx = \int_1^3 \left(\frac{3}{2} \frac{2x-4}{x^2 - 4x + 5} + 8 \cdot \frac{1}{x^2 - 4x + 5} \right) dx$$

$$= \frac{3}{2} \int_1^3 \frac{2x-4}{x^2 - 4x + 5} dx + 8 \int_1^3 \frac{1}{x^2 - 4x + 5} dx$$

$$+ \text{Xét: } M = \frac{3}{2} \int_1^3 \frac{2x-4}{x^2 - 4x + 5} dx = \frac{3}{2} \int_1^3 \frac{d(x^2 - 4x + 5)}{x^2 - 4x + 5} = \frac{3}{2} \ln |x^2 - 4x + 5| \Big|_1^3 = \frac{3}{2} (\ln 2 - \ln 2) = 0$$

$$+ \text{Xét: } N = 8 \int_1^3 \frac{1}{x^2 - 4x + 5} dx = 8 \int_1^3 \frac{dx}{(x-2)^2 + 1}$$

$$\text{Đặt: } x - 2 = \tan t \quad \text{Với } \left(t \in \left(-\frac{\pi}{2}; \frac{\pi}{2} \right) \right)$$

$$\Rightarrow dx = \frac{dt}{\cos^2 t}$$

$$\text{Đổi cận: Với } x = 1 \Rightarrow t = -\frac{\pi}{4}; \text{ Với } x = 3 \Rightarrow t = \frac{\pi}{4}$$

$$\Rightarrow N = 8 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dt}{\cos^2 t (\tan^2 t + 1)} = 8 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dt = 8t \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 4\pi$$

Vậy, $I_2 = M + N = 4\pi$

$$\text{c) } I_3 = \int_0^1 \frac{3x-1}{4x^2-4x+2} dx$$

Chú ý: $(4x^2 - 4x + 2)' = 8x - 4$

$$\text{Ta có: } I_3 = \int_0^1 \frac{3x-1}{4x^2-4x+2} dx = \int_0^1 \frac{\frac{3}{8}(8x-4) + \frac{1}{2}}{4x^2-4x+2} dx$$

$$= \frac{3}{8} \int_0^1 \frac{8x-4}{4x^2-4x+2} dx + \frac{1}{2} \int_0^1 \frac{dx}{4x^2-4x+2}$$

$$\text{+) Xét: } M = \frac{3}{8} \int_0^1 \frac{8x-4}{4x^2-4x+2} dx = \frac{3}{8} \int_0^1 \frac{d(4x^2-4x+2)}{4x^2-4x+2} = \frac{3}{8} \ln|4x^2-4x+2| \Big|_0^1 = \frac{3}{8} (\ln 2 - \ln 2) = 0$$

$$\text{+) Xét: } N = \frac{1}{2} \int_0^1 \frac{dx}{4x^2-4x+2} = \frac{1}{2} \int_0^1 \frac{dx}{(2x-1)^2+1}$$

Đặt: $2x-1 = \tan t$ Với $\left(t \in \left(-\frac{\pi}{2}; \frac{\pi}{2} \right) \right)$

$$\Rightarrow 2dx = \frac{dt}{\cos^2 t} \Leftrightarrow dx = \frac{dt}{2\cos^2 t}$$

Đổi cận: Với $x=0 \Rightarrow t = -\frac{\pi}{4}$; Với $x=1 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow N = \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dt}{2\cos^2 t (\tan^2 t + 1)} = \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dt = \frac{1}{2} t \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4}$$

Vậy, $I_3 = M + N = \frac{\pi}{4}$

HT 11. Tính các tích phân sau:

$$\text{a) } I_1 = \int_{-1}^0 \frac{x^3 - 5x^2 + 6x - 1}{x^2 - 3x + 2} dx$$

$$\text{b) } I_2 = \int_0^1 \frac{x^4 + 5x^3 - 3x^2 + 2x - 1}{x^2 + 2x + 1} dx$$

$$\text{c) } I_3 = \int_{-1}^0 \frac{x^3 + 3x^2 - 6x + 1}{x^2 + 2x + 2} dx$$

$$\text{d) } I = \int_1^2 \frac{x^2}{x^2 - 7x + 12} dx$$

Giải

$$\text{a) } I_1 = \int_{-1}^0 \frac{x^3 - 5x^2 + 6x - 1}{x^2 - 3x + 2} dx = \int_{-1}^0 \left(x - 2 + \frac{-2x + 3}{x^2 - 3x + 2} \right) dx = \int_{-1}^0 (x - 2) dx + \int_{-1}^0 \frac{-2x + 3}{x^2 - 3x + 2} dx$$

$$\text{+) Xét: } M = \int_{-1}^0 (x - 2) dx = \left(\frac{x^2}{2} - 2x \right) \Big|_{-1}^0 = -\left(\frac{1}{2} + 2 \right) = -\frac{5}{2}$$

$$\text{+) Xét: } N = \int_{-1}^0 \frac{-2x + 3}{x^2 - 3x + 2} dx = \int_{-1}^0 \frac{-2x + 3}{(x - 1)(x - 2)} dx$$

Dùng đồng nhất thức ta tách được:

$$N = \int_{-1}^0 \left(\frac{-1}{x - 1} + \frac{-1}{x - 2} \right) dx = \left(-\ln|x - 1| - \ln|x - 2| \right) \Big|_{-1}^0 = (-\ln 1 - \ln 2) - (-\ln 2 - \ln 3) = \ln 3$$

$$\text{Vậy, } I_1 = M + N = \ln 3 - \frac{5}{2}$$

$$\text{b) } I_2 = \int_0^1 \frac{x^4 + 5x^3 - 3x^2 + 2x - 1}{x^2 + 2x + 1} dx = \int_0^1 \left(x^2 + 3x - 10 + \frac{19x + 9}{x^2 + 2x + 1} \right) dx$$

$$\text{+) Xét: } M = \int_0^1 (x^2 + 3x - 10) dx = \left(\frac{x^3}{3} + \frac{3x^2}{2} - 10x \right) \Big|_0^1 = \left(\frac{1}{3} + \frac{3}{2} - 10 \right) - 0 = -\frac{49}{6}$$

$$\text{+) Xét: } N = \int_0^1 \frac{19x + 9}{x^2 + 2x + 1} dx = \int_0^1 \frac{19(x + 1) - 10}{(x + 1)^2} dx = \int_0^1 \left(\frac{19}{x + 1} - \frac{10}{(x + 1)^2} \right) dx$$

$$= \left(19 \ln|x + 1| + \frac{10}{x + 1} \right) \Big|_0^1 = (19 \ln 2 + 5) - (19 \ln 1 + 10) = 19 \ln 2 - 5$$

$$\text{Vậy, } I_2 = M + N = 19 \ln 2 - \frac{79}{6}$$

$$\text{c) } I_3 = \int_{-1}^0 \frac{x^3 + 3x^2 - 6x + 1}{x^2 + 2x + 2} dx = \int_{-1}^0 \left(x + 1 - \frac{10x + 1}{x^2 + 2x + 2} \right) dx$$

$$\text{+) Xét: } M = \int_{-1}^0 (x + 1) dx = \left(\frac{x^2}{2} + x \right) \Big|_{-1}^0 = -\left(\frac{1}{2} - 1 \right) = \frac{1}{2}$$

$$\text{+) Xét: } N = \int_{-1}^0 \frac{10x + 1}{x^2 + 2x + 2} dx = \int_{-1}^0 \frac{5(2x + 2) - 9}{x^2 + 2x + 2} dx = \int_{-1}^0 \left(\frac{5(2x + 2)}{x^2 + 2x + 2} - \frac{9}{x^2 + 2x + 2} \right) dx$$

$$P = 5 \int_{-1}^0 \frac{2x + 2}{x^2 + 2x + 2} dx = 5 \int_{-1}^0 \frac{d(x^2 + 2x + 2)}{x^2 + 2x + 2} = 5 \ln|x^2 + 2x + 2| \Big|_{-1}^0 = 5(\ln 2 - \ln 1) = 5 \ln 2$$

$$Q = 9 \int_{-1}^0 \frac{dx}{x^2 + 2x + 2} = 9 \int_{-1}^0 \frac{dx}{(x+1)^2 + 1}$$

Đặt: $x + 1 = \tan t$ Với $t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$$\Rightarrow dx = \frac{dt}{\cos^2 t}$$

Đổi cận: Với $x = -1 \Rightarrow t = 0$; Với $x = 0 \Rightarrow t = \frac{\pi}{4}$

$$\Rightarrow Q = 9 \int_0^{\frac{\pi}{4}} \frac{dt}{\cos^2 t (\tan^2 t + 1)} = 9 \int_0^{\frac{\pi}{4}} dt = 9t \Big|_0^{\frac{\pi}{4}} = \frac{9\pi}{4}$$

$$\Rightarrow N = P - Q = 5 \ln 2 - \frac{9\pi}{4} \Rightarrow I_3 = M + N = \frac{1}{2} + 5 \ln 2 - \frac{9\pi}{4}$$

d) $I = \int_1^2 \left(1 + \frac{16}{x-4} - \frac{9}{x-3}\right) dx = (x + 16 \ln|x-4| - 9 \ln|x-3|) \Big|_1^2 = 1 + 25 \ln 2 - 16 \ln 3.$

HT 12. Tính các tích phân sau:

a) $I = \int_1^2 \frac{dx}{x^5 + x^3}$ b) $I = \int_0^1 \frac{x dx}{(x+1)^3}$

Giải

a) $I = \int_1^2 \frac{dx}{x^5 + x^3}$

Ta có: $\frac{1}{x^3(x^2+1)} = -\frac{1}{x} + \frac{1}{x^3} + \frac{x}{x^2+1}$

$$\Rightarrow I = \left[-\ln|x| - \frac{1}{2x^2} + \frac{1}{2} \ln(x^2+1) \right] \Big|_1^2 = -\frac{3}{2} \ln 2 + \frac{1}{2} \ln 5 + \frac{3}{8}$$

b) $I = \int_0^1 \frac{x dx}{(x+1)^3}$

Ta có: $\frac{x}{(x+1)^3} = \frac{x+1-1}{(x+1)^3} = (x+1)^{-2} - (x+1)^{-3}$

$$\Rightarrow I = \int_0^1 [(x+1)^{-2} - (x+1)^{-3}] dx = \frac{1}{8}$$

LƯU HUY THƯỜNG

HT 13. Tính các tích phân sau: (Đổi biến số)

$$1. I = \int_0^1 \frac{x^7}{(1+x^2)^5} dx$$

$$2. I = \int_0^1 x^5(1-x^3)^6 dx$$

$$3. I = \int_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)} dx$$

$$4. I = \int_1^2 \frac{dx}{x \cdot (x^{10}+1)^2}$$

$$5. I = \int_1^2 \frac{1-x^7}{x(1+x^7)} dx$$

$$6. I = \int_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)}$$

$$7. I = \int_0^1 \frac{(x-1)^2}{(2x+1)^4} dx$$

$$8. I = \int_0^1 \frac{(7x-1)^{99}}{(2x+1)^{101}} dx$$

$$9. I = \int_1^2 \frac{1+x^2}{1+x^4} dx$$

$$10. I = \int_1^2 \frac{1-x^2}{1+x^4} dx$$

$$11. I = \int_1^2 \frac{1-x^2}{x+x^3} dx$$

$$12. I = \int_0^1 \frac{x^4+1}{x^6+1} dx$$

$$13. I = \int_0^{\sqrt{3}} \frac{x^2}{x^4-1} dx$$

$$14. I = \int_0^1 \frac{x dx}{x^4+x^2+1}$$

$$15. I = \int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1} dx$$

Bài giải

$$1. I = \int_0^1 \frac{x^7}{(1+x^2)^5} dx = \int_0^1 \frac{(x^2)^3 x dx}{(1+x^2)^5}$$

$$\text{Đặt } t = 1+x^2 \Rightarrow dt = 2x dx$$

$$\text{Đổi cận: Với } x=0 \Rightarrow t=1; \text{ Với } x=1 \Rightarrow t=2$$

$$\Rightarrow I = \frac{1}{2} \int_1^2 \frac{(t-1)^3}{t^5} dt = \frac{1}{4} \cdot \frac{1}{2^5}$$

$$2. I = \int_0^1 x^5(1-x^3)^6 dx = \int_0^1 x^3(1-x^3)x^2 dx$$

$$\text{Đặt } t = 1-x^3 \Rightarrow dt = -3x^2 dx \Rightarrow x^2 dx = -\frac{dt}{3}$$

$$\text{Đổi cận: Với } x=0 \Rightarrow t=1; \text{ Với } x=1 \Rightarrow t=0$$

$$\Rightarrow I = \frac{1}{3} \int_1^0 t^6(1-t) dt = \frac{1}{3} \left(\frac{t^7}{7} - \frac{t^8}{8} \right) = \frac{1}{168}$$

$$3. I = \int_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)} dx = \int_1^{\sqrt[4]{3}} \frac{x^3 dx}{x^4(x^4+1)}$$

$$\text{Đặt } t = x^4 \Rightarrow dt = 4x^3 dx \Rightarrow x^3 dx = \frac{dt}{4}$$

$$\text{Đổi cận: Với } x = 1 \Rightarrow t = 1; \text{ Với } x = \sqrt[4]{3} \Rightarrow t = 3$$

$$\Rightarrow I = \frac{1}{4} \int_1^3 \frac{dt}{t(t+1)} = \frac{1}{4} \int_1^3 \left(\frac{1}{t} - \frac{1}{t+1} \right) dt = \frac{1}{4} \ln \left(\frac{t}{t+1} \right) \Big|_1^3 = \frac{1}{4} \ln \frac{3}{2}$$

$$4. I = \int_1^2 \frac{dx}{x(x^{10} + 1)^2} = \int_1^2 \frac{x^9 dx}{x^{10}(x^{10} + 1)^2}$$

$$\text{Đặt } t = x^{10} + 1 \Rightarrow dt = 10x^9 dx \Rightarrow x^9 dx = \frac{dt}{10}$$

$$\text{Đổi cận: Với } x = 1 \Rightarrow t = 2; \text{ Với } x = 2 \Rightarrow t = 2^{10} + 1$$

$$\Rightarrow I = \frac{1}{5} \int_2^{2^{10}+1} \frac{dt}{(t-1)t^2} = \frac{1}{5} \int_2^{2^{10}+1} \left(\frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2} \right) dt$$

$$= \frac{1}{5} \left(\ln(t-1) - \ln t + \frac{1}{t} \right) \Big|_2^{2^{10}+1} = \frac{1}{5} (10 \ln 2 - \ln(2^{10} + 1) + \frac{1}{2^{10} + 1}) - \frac{1}{5} (-\ln 2 + \frac{1}{2})$$

$$5. I = \int_1^2 \frac{1-x^7}{x(1+x^7)} dx = \int_1^2 \frac{(1-x^7) \cdot x^6}{x^7 \cdot (1+x^7)} dx.$$

$$\text{Đặt } t = x^7 \Rightarrow dt = 7x^6 dx \Rightarrow x^6 dx = \frac{dt}{7}$$

$$\text{Đổi cận: Với } x = 1 \Rightarrow t = 1; \text{ Với } x = 2 \Rightarrow t = 128$$

$$\Rightarrow I = \frac{1}{7} \int_1^{128} \frac{1-t}{t(1+t)} dt = \frac{1}{7} \int_1^{128} \left(\frac{1}{t} - \frac{2}{1+t} \right) dt = \frac{1}{7} (\ln|t| - 2 \ln|1+t|) \Big|_1^{128}$$

$$= \frac{1}{7} (7 \ln 2 - 2 \ln 129) - \frac{1}{7} (-2 \ln 2) = \frac{10}{7} \ln 2 - \frac{2}{7} \ln 129$$

$$6. I = \int_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)} = \int_1^{\sqrt{3}} \frac{dx}{x^2 \cdot x^6 \left(\frac{1}{x^2} + 1 \right)}$$

$$\text{Đặt } t = \frac{1}{x} \Rightarrow dt = -\frac{1}{x^2} dx$$

$$: \text{Đổi cận: Với } x = 1 \Rightarrow t = 1; \text{ Với } x = \sqrt{3} \Rightarrow t = \frac{1}{\sqrt{3}}$$

$$\Rightarrow I = - \int_1^{\frac{1}{\sqrt{3}}} \frac{t^6}{t^2+1} dt = \int_{\frac{1}{\sqrt{3}}}^1 \left(t^4 - t^2 + 1 - \frac{1}{t^2+1} \right) dt = \frac{117 - 41\sqrt{3}}{135} + \frac{\pi}{12}$$

$$7. I = \int_0^1 \frac{(x-1)^2}{(2x+1)^4} dx = \int_0^1 \left(\frac{x-1}{2x+1} \right)^2 \frac{dx}{(2x+1)^2}$$

Chú ý: $\left(\frac{x-1}{2x+1}\right)' = \frac{3}{(2x+1)^2}$

Đặt: $\frac{x-1}{2x+1} = t \Rightarrow \frac{3dx}{(2x+1)^2} = dt \Rightarrow \frac{dx}{(2x+1)^2} = \frac{dt}{3}$

Đổi cận: Với: $x = 0 \Rightarrow t = -1$; Với $x = 1 \Rightarrow t = 0$

$$\Rightarrow t = \frac{1}{3} \int_0^{-1} t^2 dt = \frac{t^3}{9} \Big|_0^{-1} = -\frac{1}{9}$$

$$\begin{aligned} 8. I &= \int_0^1 \left(\frac{7x-1}{2x+1}\right)^{99} \frac{dx}{(2x+1)^2} = \frac{1}{9} \int_0^1 \left(\frac{7x-1}{2x+1}\right)^{99} d\left(\frac{7x-1}{2x+1}\right) \\ &= \frac{1}{9} \cdot \frac{1}{100} \left(\frac{7x-1}{2x+1}\right)^{100} \Big|_0^1 = \frac{1}{900} [2^{100} - 1] \end{aligned}$$

$$9. I = \int_1^2 \frac{1+x^2}{1+x^4} dx$$

Ta có: $\frac{1+x^2}{1+x^4} = \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}}$.

Đặt $t = x - \frac{1}{x} \Rightarrow dt = \left(1 + \frac{1}{x^2}\right) dx$

Đổi cận: Với $x = 1 \Rightarrow t = 0$; Với $x = 2 \Rightarrow t = \frac{3}{2}$

$$\Rightarrow I = \int_0^{\frac{3}{2}} \frac{dt}{t^2 - 2} = \frac{1}{2\sqrt{2}} \int_0^{\frac{3}{2}} \left(\frac{1}{t - \sqrt{2}} - \frac{1}{t + \sqrt{2}} \right) dt = \frac{1}{2\sqrt{2}} \cdot \ln \left| \frac{t - \sqrt{2}}{t + \sqrt{2}} \right| \Big|_0^{\frac{3}{2}} = \frac{1}{\sqrt{2}} \ln(3 - 2\sqrt{2})$$

$$10. I = \int_1^2 \frac{1-x^2}{1+x^4} dx$$

Ta có: $\frac{1-x^2}{1+x^4} = \frac{\frac{1}{x^2} - 1}{x^2 + \frac{1}{x^2}}$.

Đặt $t = x + \frac{1}{x} \Rightarrow dt = \left(1 - \frac{1}{x^2}\right) dx$

Đổi cận: Với $x = 1 \Rightarrow t = 2$; Với $x = 2 \Rightarrow t = \frac{5}{2}$

$$\Rightarrow I = -\int_2^{\frac{5}{2}} \frac{dt}{t^2 + 2}.$$

$$\text{Đặt } t = \sqrt{2} \tan u \Rightarrow dt = \sqrt{2} \frac{du}{\cos^2 u}; \tan u = 2 \Rightarrow u_1 = \arctan 2; \tan u = \frac{5}{2} \Rightarrow u_2 = \arctan \frac{5}{2}$$

$$\Rightarrow I = \frac{\sqrt{2}}{2} \int_{u_1}^{u_2} du = \frac{\sqrt{2}}{2} (u_2 - u_1) = \frac{\sqrt{2}}{2} \left(\arctan \frac{5}{2} - \arctan 2 \right)$$

$$11. I = \int_1^2 \frac{1-x^2}{x+x^3} dx$$

$$\text{Ta có: } I = \int_1^2 \frac{\frac{1}{x^2} - 1}{\frac{1}{x} + x} dx. \text{ Đặt } t = x + \frac{1}{x} \Rightarrow dt = \left(1 - \frac{1}{x^2} \right) dx$$

$$\text{Đổi cận: Với } x = 1 \Rightarrow t = 2; \text{ Với } x = 2 \Rightarrow t = \frac{5}{2}$$

$$I = -\int_2^{\frac{5}{2}} \frac{dt}{t} = -\ln t \Big|_2^{\frac{5}{2}} = -\ln \frac{5}{2} + \ln 2 = \ln \frac{4}{5}$$

$$12. I = \int_0^1 \frac{x^4 + 1}{x^6 + 1} dx$$

$$\text{Ta có: } \frac{x^4 + 1}{x^6 + 1} = \frac{(x^4 - x^2 + 1) + x^2}{x^6 + 1} = \frac{x^4 - x^2 + 1}{(x^2 + 1)(x^4 - x^2 + 1)} + \frac{x^2}{x^6 + 1} = \frac{1}{x^2 + 1} + \frac{x^2}{x^6 + 1}$$

$$\Rightarrow I = \int_0^1 \frac{1}{x^2 + 1} dx + \frac{1}{3} \int_0^1 \frac{d(x^3)}{(x^3)^2 + 1} dx = \frac{\pi}{4} + \frac{1}{3} \cdot \frac{\pi}{4} = \frac{\pi}{3}$$

$$13. I = \int_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{x^4 - 1} dx$$

$$I = \int_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{(x^2 - 1)(x^2 + 1)} dx = \frac{1}{2} \int_0^{\frac{\sqrt{3}}{3}} \left(\frac{1}{x^2 - 1} + \frac{1}{x^2 + 1} \right) dx = \frac{1}{4} \ln(2 - \sqrt{3}) + \frac{\pi}{12}$$

$$14. I = \int_0^1 \frac{xdx}{x^4 + x^2 + 1}$$

$$\text{Đặt } t = x^2 \Rightarrow dt = 2xdx \Rightarrow xdx = \frac{dt}{2}$$

Đổi cận: $x = 0 \Rightarrow t = 0$; Với $x = 1 \Rightarrow t = 1$

$$\Rightarrow I = \frac{1}{2} \int_0^1 \frac{dt}{t^2 + t + 1} = \frac{1}{2} \int_0^1 \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\pi}{6\sqrt{3}}$$

$$15. I = \int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2 + 1}{x^4 - x^2 + 1} dx$$

Ta có: $\frac{x^2 + 1}{x^4 - x^2 + 1} = \frac{1 + \frac{1}{x^2}}{x^2 + \frac{1}{x^2} - 1}$.

Đặt $t = x - \frac{1}{x} \Rightarrow dt = \left(1 + \frac{1}{x^2}\right) dx$

$$\Rightarrow I = \int_0^1 \frac{dt}{t^2 + 1}$$

Đặt $t = \tan u \Rightarrow dt = \frac{du}{\cos^2 u} \Rightarrow I = \int_0^{\frac{\pi}{4}} du = \frac{\pi}{4}$

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PHẦN III TÍCH PHÂN HÀM SỐ VÔ TỶ

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HT 1. Tính các tích phân sau:

a) $I_1 = \int_0^{\sqrt{3}} \frac{x dx}{\sqrt{x^2 + 1}}$

b) $I_2 = \int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}}$

c) $I_3 = \int_0^{\sqrt{3}} \sqrt{x^2 + 1} dx$

Bài giải

a) $I_1 = \int_0^{\sqrt{3}} \frac{x dx}{\sqrt{x^2 + 1}} = \frac{1}{2} \int_0^{\sqrt{3}} \frac{d(x^2 + 1)}{\sqrt{x^2 + 1}} = \sqrt{x^2 + 1} \Big|_0^{\sqrt{3}} = \sqrt{2}$

b) $I_2 = \int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}}$

Đặt: $x + \sqrt{x^2 + 1} = t \Rightarrow (1 + \frac{x}{\sqrt{x^2 + 1}}) dx = dt \Leftrightarrow \frac{x + \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}} dx = dt \Leftrightarrow \frac{dx}{\sqrt{x^2 + 1}} = \frac{dt}{t}$

Đổi cận: $x = 0 \Rightarrow t = 1; x = \sqrt{3} \Rightarrow t = \sqrt{3} + 2$

$$\Rightarrow I_2 = \int_1^{\sqrt{3}+2} \frac{dt}{t} = \ln|t| \Big|_1^{\sqrt{3}+2} = \ln(\sqrt{3} + 2)$$

c) $I_3 = \int_0^{\sqrt{3}} \sqrt{x^2 + 1} dx$

Đặt: $\begin{cases} u = \sqrt{x^2 + 1} \\ dv = dx \end{cases} \Rightarrow \begin{cases} du = \frac{x}{\sqrt{x^2 + 1}} dx \\ v = x \end{cases}$

$$\Rightarrow I_3 = x\sqrt{x^2 + 1} \Big|_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{x^2 dx}{\sqrt{x^2 + 1}} = 2\sqrt{3} - \int_0^{\sqrt{3}} \frac{x^2 + 1 - 1}{\sqrt{x^2 + 1}} dx$$

$$= 2\sqrt{3} - \int_0^{\sqrt{3}} \sqrt{x^2 + 1} dx + \int_0^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}} = 2\sqrt{3} - I_3 + I_2 = 2\sqrt{3} - I_3 + \ln(\sqrt{3} + 2)$$

$$\Rightarrow 2I_3 = 2\sqrt{3} + \ln(\sqrt{3} + 2) \Rightarrow I_3 = \sqrt{3} + \frac{1}{2} \ln(\sqrt{3} + 2)$$

HT 2. Tính các tích phân sau:

a) $I = \int_0^1 x^3 \sqrt{1-x^2} dx$

b) $I = \int_{-1}^0 x \sqrt[3]{x+1} dx$

c) $I = \int_0^1 (x-1)^3 \sqrt{2x-x^2} dx$

Bài giải

a) $I = \int_0^1 x^3 \sqrt{1-x^2} dx = \int_0^1 x^2 \sqrt{1-x^2} x dx$

Đặt: $t = \sqrt{1-x^2} (t \geq 0) \Leftrightarrow x^2 = 1-t^2 \Rightarrow x dx = -t dt$

Đổi cận: $x = 0 \Rightarrow t = 1; x = 1 \Rightarrow t = 0$

$$\Rightarrow I = - \int_1^0 (1-t^2)t \cdot t dt = \int_0^1 (t^2 - t^4) dt = \left(\frac{t^3}{3} - \frac{t^5}{5} \right) \Big|_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

b) $I = \int_{-1}^0 x \sqrt[3]{x+1} dx$

Đặt $t = \sqrt[3]{x+1} \Rightarrow t^3 = x+1 \Rightarrow dx = 3t^2 dt$

Đổi cận: $x = -1 \Rightarrow t = 0; x = 0 \Rightarrow t = 1$

$$\Rightarrow I = \int_0^1 3(t^3 - 1) dt = 3 \left(\frac{t^7}{7} - \frac{t^4}{4} \right) \Big|_0^1 = -\frac{9}{28}$$

c) $I = \int_0^1 (x-1)^3 \sqrt{2x-x^2} dx$

$$I = \int_0^1 (x-1)^3 \sqrt{2x-x^2} dx = \int_0^1 (x^2 - 2x + 1) \sqrt{2x-x^2} (x-1) dx.$$

Đặt $t = \sqrt{2x-x^2} \Leftrightarrow t^2 = 2x-x^2 \Rightarrow 2t dt = (2-2x) dx \Leftrightarrow (x-1) dx = -t dt$

Đổi cận: $x = 0 \Rightarrow t = 0; x = 1 \Rightarrow t = 1$

$$\Rightarrow I = - \int_0^1 (-t^2 + 1) t \cdot t dt = \int_0^1 (t^4 - t^2) dt = \left(\frac{t^5}{5} - \frac{t^3}{3} \right) \Big|_0^1 = \frac{1}{5} - \frac{1}{3} = -\frac{2}{15}.$$

HT 3. Tính các tích phân sau:

a) $I = \int_0^4 \frac{\sqrt{2x+1}}{1+\sqrt{2x+1}} dx$

b) $I = \int_2^6 \frac{dx}{2x+1+\sqrt{4x+1}}$

c) $I = \int_0^1 \frac{1+x}{1+\sqrt{x}} dx$

d) $I = \int_0^3 \frac{x-3}{3\sqrt{x+1}+x+3} dx$

e) $I = \int_1^5 \frac{x^2+1}{x\sqrt{3x+1}} dx$

f) $I = \int_0^3 \frac{2x^2+x-1}{\sqrt{x+1}} dx$

g) $I = \int_0^1 \frac{x^2 dx}{(x+1)\sqrt{x+1}}$

h) $I = \int_0^4 \frac{x+1}{(1+\sqrt{1+2x})^2} dx$

i) $I = \int_0^2 \frac{2x^3-3x^2+x}{\sqrt{x^2-x+1}} dx$

j) $I = \int_0^2 \frac{x^3 dx}{\sqrt[3]{4+x^2}}$

k) $I = \int_1^2 \frac{\sqrt{4-x^2}}{x} dx$

l) $I = \int_2^{2\sqrt{5}} \frac{x}{(x^2+1)\sqrt{x^2+5}} dx$

m) $I = \int_1^{27} \frac{\sqrt{x}-2}{x+\sqrt[3]{x^2}} dx$

o) $I = \int_{\sqrt[3]{3}}^{\sqrt[3]{8}} \frac{x-1}{\sqrt{x^2+1}} dx$

p) $I = \int_1^4 \frac{x^2+\sqrt{x}}{\sqrt{1+x\sqrt{x}}} dx$

Bài giải

a) $I = \int_0^4 \frac{\sqrt{2x+1}}{1+\sqrt{2x+1}} dx$

Đặt $t = \sqrt{2x+1} \Rightarrow t^2 = 2x+1 \Rightarrow 2tdt = 2dx \Leftrightarrow dx = tdt$

Đổi cận: $x=0 \Rightarrow t=1; x=4 \Rightarrow t=3$

$$\Rightarrow I = \int_1^3 \frac{t^2}{1+t} dt = \int_1^3 \left(t - 1 + \frac{1}{t+1} \right) dt = \left(\frac{t^2}{2} - t + \ln|t+1| \right) \Big|_1^3$$

$$= \left(\frac{9}{2} - 3 + \ln 4 \right) - \left(\frac{1}{2} - 1 + \ln 2 \right) = 2 + \ln 2$$

b) $I = \int_2^6 \frac{dx}{2x+1+\sqrt{4x+1}}$

Đặt $t = \sqrt{4x+1} \Rightarrow t^2 = 4x+1 \Rightarrow 2tdt = 4dx \Rightarrow dx = \frac{tdt}{2}$

Đổi cận: $x=2 \Rightarrow t=3; x=6 \Rightarrow t=5$

$$\Rightarrow I = \frac{1}{2} \int_3^5 \frac{tdt}{\frac{t^2-1}{2} + 1 + t} = \int_3^5 \frac{tdt}{t^2+2t+1} = \int_3^5 \frac{tdt}{(t+1)^2} = \int_3^5 \left(\frac{1}{t+1} - \frac{1}{(t+1)^2} \right) dt$$

$$= \left(\ln|t+1| + \frac{1}{t+1} \right) \Big|_3^5 = \left(\ln 6 + \frac{1}{6} \right) - \left(\ln 4 + \frac{1}{4} \right) = \ln \frac{3}{2} - \frac{1}{12}$$

c) $I = \int_0^1 \frac{1+x}{1+\sqrt{x}} dx$

Đặt $t = 1 + \sqrt{x} \Rightarrow x = (t-1)^2 \Rightarrow dx = 2(t-1)dt$

Đổi cận: $x=0 \Rightarrow t=1; x=1 \Rightarrow t=2$

$$\Rightarrow I = \int_1^2 \frac{1+(t-1)^2}{t} dt = \int_1^2 \left(t-2 + \frac{2}{t} \right) dt = \left(\frac{t^2}{2} - 2t + 2 \ln|t| \right) \Big|_1^2 = \frac{11}{3} - 4 \ln 2.$$

$$d) I = \int_0^3 \frac{x-3}{3\sqrt{x+1} + x+3} dx$$

$$\text{Đặt } t = \sqrt{x+1} \Rightarrow 2t dt = dx$$

$$\text{Đổi cận: } x=0 \Rightarrow t=1; x=3 \Rightarrow t=2$$

$$\Rightarrow I = \int_1^2 \frac{2t^3 - 8t}{t^2 + 3t + 2} dt = \int_1^2 (2t-6) dt + 6 \int_1^2 \frac{1}{t+1} dt = -3 + 6 \ln \frac{3}{2}$$

$$e) I = \int_1^5 \frac{x^2 + 1}{x\sqrt{3x+1}} dx$$

$$\text{Đặt } t = \sqrt{3x+1} \Rightarrow dx = \frac{2tdt}{3}$$

$$\text{Đổi cận: } x=1 \Rightarrow t=2; x=5 \Rightarrow t=4$$

$$\begin{aligned} \Rightarrow I &= \int_2^4 \frac{\left(\frac{t^2-1}{3}\right)^2 + 1}{\frac{t^2-1}{3} \cdot t} \cdot \frac{2tdt}{3} = \frac{2}{9} \int_2^4 (t^2-1) dt + 2 \int_2^4 \frac{dt}{t^2-1} = \frac{2}{9} \int_2^4 (t^2-1) dt + 2 \int_2^4 \left(\frac{1}{t-1} - \frac{1}{t+1} \right) dt \\ &= \frac{2}{9} \left(\frac{1}{3} t^3 - t \right) \Big|_2^4 + \ln \left| \frac{t-1}{t+1} \right| \Big|_2^4 = \frac{100}{27} + \ln \frac{9}{5}. \end{aligned}$$

$$f) I = \int_0^3 \frac{2x^2 + x - 1}{\sqrt{x+1}} dx$$

$$\text{Đặt } \sqrt{x+1} = t \Leftrightarrow x = t^2 - 1 \Rightarrow dx = 2t dt$$

$$\text{Đổi cận: } x=0 \Rightarrow t=1; x=3 \Rightarrow t=2$$

$$\Rightarrow I = \int_1^2 \frac{2(t^2-1)^2 + (t^2-1) - 1}{t} 2t dt = 2 \int_1^2 (2t^4 - 3t^2) dt = \left(\frac{4t^5}{5} - 2t^3 \right) \Big|_1^2 = \frac{54}{5}$$

$$g) I = \int_0^1 \frac{x^2 dx}{(x+1)\sqrt{x+1}}$$

$$\text{Đặt } t = \sqrt{x+1} \Rightarrow t^2 = x+1 \Rightarrow 2tdt = dx$$

$$\text{Đổi cận: } x=0 \Rightarrow t=1; x=1 \Rightarrow t=\sqrt{2}$$

$$\Rightarrow I = \int_1^{\sqrt{2}} \frac{(t^2-1)^2}{t^3} \cdot 2tdt = 2 \int_1^{\sqrt{2}} \left(t - \frac{1}{t}\right)^2 dt = 2 \left(\frac{t^3}{3} - 2t - \frac{1}{t} \right) \Big|_1^{\sqrt{2}} = \frac{16-11\sqrt{2}}{3}$$

$$h) I = \int_0^4 \frac{x+1}{(1+\sqrt{1+2x})^2} dx$$

$$\text{Đặt } t = 1 + \sqrt{1+2x} \Rightarrow dt = \frac{dx}{\sqrt{1+2x}} \Rightarrow dx = (t-1)dt \text{ và } x = \frac{t^2-2t}{2}$$

$$\text{Đổi cận: } x=0 \Rightarrow t=2; x=4 \Rightarrow t=4$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{2} \int_2^4 \frac{(t^2-2t+2)(t-1)}{t^2} dt = \frac{1}{2} \int_2^4 \frac{t^3-3t^2+4t-2}{t^2} dt = \frac{1}{2} \int_2^4 \left(t-3+\frac{4}{t}-\frac{2}{t^2} \right) dt \\ &= \frac{1}{2} \left(\frac{t^2}{2} - 3t + 4 \ln|t| + \frac{2}{t} \right) \Big|_2^4 = 2 \ln 2 - \frac{1}{4} \end{aligned}$$

$$i) I = \int_0^2 \frac{2x^3-3x^2+x}{\sqrt{x^2-x+1}} dx = \int_0^2 \frac{(x^2-x)(2x-1)}{\sqrt{x^2-x+1}} dx$$

$$\text{Đặt } t = \sqrt{x^2-x+1} \Rightarrow 2tdt = (2x-1)dx$$

$$\text{Đổi cận: } x=0 \Rightarrow t=1; x=2 \Rightarrow t=\sqrt{3}$$

$$\Rightarrow I = 2 \int_1^{\sqrt{3}} (t^2-1)dt = \frac{4}{3}$$

$$j) I = \int_0^2 \frac{x^3 dx}{\sqrt[3]{4+x^2}}$$

$$\text{Đặt } t = \sqrt[3]{4+x^2} \Rightarrow x^2 = t^3-4 \Rightarrow 2xdx = 3t^2 dt$$

$$\text{Đổi cận: } x=0 \Rightarrow t=\sqrt[3]{4}; x=2 \Rightarrow t=2$$

$$\Rightarrow I = \frac{3}{2} \int_{\sqrt[3]{4}}^2 (t^4-4t)dt = \frac{3}{2} \left(\frac{t^5}{5} - 2t^2 \right) \Big|_{\sqrt[3]{4}}^2 = -\frac{3}{2} \left(\frac{8}{5} + 4\sqrt[3]{2} \right)$$

$$k) I = \int_1^2 \frac{\sqrt{4-x^2}}{x} dx$$

$$\text{Ta có: } I = \int_1^2 \frac{\sqrt{4-x^2}}{x^2} x dx.$$

$$\text{Đặt } t = \sqrt{4-x^2} \Rightarrow t^2 = 4-x^2 \Rightarrow t dt = -x dx$$

$$\text{Đổi cận: } x=1 \Rightarrow t=\sqrt{3}; x=2 \Rightarrow t=0$$

$$\Rightarrow I = \int_{\sqrt{3}}^0 \frac{t(-tdt)}{4-t^2} = \int_{\sqrt{3}}^0 \frac{t^2}{t^2-4} dt = \int_{\sqrt{3}}^0 \left(1 + \frac{4}{t^2-4}\right) dt = \left(t + \ln \left| \frac{t-2}{t+2} \right| \right) \Big|_{\sqrt{3}}^0 = -\left(\sqrt{3} + \ln \left| \frac{2-\sqrt{3}}{2+\sqrt{3}} \right| \right)$$

$$l) I = \int_2^{2\sqrt{5}} \frac{x}{(x^2+1)\sqrt{x^2+5}} dx$$

$$\text{Đặt } t = \sqrt{x^2+5} \Rightarrow t^2 = x^2+5 \Rightarrow t dt = x dx$$

$$\text{Đổi cận: } x=2 \Rightarrow t=3; x=2\sqrt{5} \Rightarrow t=5$$

$$I = \int_3^5 \frac{dt}{t^2-4} = \frac{1}{4} \int_3^5 \left(\frac{1}{t-2} - \frac{1}{t+2} \right) dt = \frac{1}{4} \ln \frac{15}{7}.$$

$$m) I = \int_1^{27} \frac{\sqrt{x}-2}{x+\sqrt[3]{x^2}} dx$$

$$\text{Đặt } t = \sqrt[6]{x} \Rightarrow t^6 = x \Rightarrow dx = 6t^5 dt$$

$$\text{Đổi cận: } x=1 \Rightarrow t=1; x=27 \Rightarrow t=\sqrt{3}$$

$$\Rightarrow I = 5 \int_1^{\sqrt{3}} \frac{t^3-2}{t(t^2+1)} dt = 5 \int_1^{\sqrt{3}} \left[1 - \frac{2}{t} + \frac{2t}{t^2+1} - \frac{1}{t^2+1} \right] dt = 5 \left(\sqrt{3} - 1 + \ln \frac{2}{3} \right) - \frac{5\pi}{12}$$

$$o) I = \int_{\sqrt{3}}^{\sqrt{8}} \frac{x-1}{\sqrt{x^2+1}} dx$$

$$I = \int_{\sqrt{3}}^{\sqrt{8}} \left(\frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}} \right) dx = \frac{1}{2} \int_{\sqrt{3}}^{\sqrt{8}} \frac{d(x^2+1)}{\sqrt{x^2+1}} - \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{\sqrt{x^2+1}}$$

$$= \left[\sqrt{x^2+1} - \ln(x + \sqrt{x^2+1}) \right] \Big|_{\sqrt{3}}^{\sqrt{8}} = 1 + \ln(\sqrt{3}+2) - \ln(\sqrt{8}+3)$$

$$p) I = \int_0^4 \frac{x^2 + \sqrt{x}}{\sqrt{1+x\sqrt{x}}} dx$$

$$\int_1^4 \frac{x^2 + \sqrt{x}}{\sqrt{1+x\sqrt{x}}} dx = \int_0^4 \frac{x^2}{\sqrt{1+x\sqrt{x}}} dx + \int_0^4 \frac{\sqrt{x}}{\sqrt{1+x\sqrt{x}}} dx$$

$$+ I_1 = \int_0^4 \frac{x^2}{\sqrt{1+x\sqrt{x}}} dx.$$

$$\text{Đặt } t = \sqrt{1+x\sqrt{x}} \Leftrightarrow t^2 - 1 = x\sqrt{x} \Leftrightarrow x^3 = (t^2 - 1)^2 \Leftrightarrow x^2 dx = \frac{4}{3} t(t^2 - 1) dt$$

$$\text{Đổi cận: } x = 0 \Rightarrow t = 1; x = 4 \Rightarrow t = 3$$

$$\Rightarrow \int_1^3 \frac{4}{3} (t^2 - 1) dt = \left(\frac{4}{9} t^3 - \frac{4}{3} t \right) \Big|_1^3 = \frac{80}{9}$$

$$+ I_2 = \int_0^4 \frac{\sqrt{x}}{\sqrt{1+x\sqrt{x}}} dx = \frac{2}{3} \int_0^4 \frac{d(1+x\sqrt{x})}{\sqrt{1+x\sqrt{x}}} = \frac{4}{3} \sqrt{1+x\sqrt{x}} \Big|_0^4 = \frac{8}{3}$$

$$\text{Vậy: } I = \frac{104}{9}$$

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HT 4. Tính các tích phân sau:

$$\text{a) } I = \int_{-1}^1 \frac{dx}{1+x+\sqrt{1+x^2}}$$

$$\text{b) } I = \int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$$

$$\text{c) } I = \int_0^1 \frac{1}{\sqrt{x^2+x+1}} dx$$

$$\text{d) } I = \int_0^3 \frac{x^2}{(1+\sqrt{1+x})^2 (2+\sqrt{1+x})^2} dx$$

$$\text{e) } I = \int_0^3 \frac{x^2}{2(x+1)+2\sqrt{x+1}+x\sqrt{x+1}} dx$$

$$\text{f) } I = \int_1^{2\sqrt{2}} \frac{\sqrt[3]{x-x^3} + 2011x}{x^4} dx$$

$$\text{g) } I = \int_{\sqrt{3}}^{2\sqrt{2}} \frac{x^4}{\left(x - \frac{1}{x}\right) \sqrt{x^2+1}} dx$$

$$\text{h) } I = \int_{\frac{1}{3}}^{\frac{2}{3}} \frac{x}{3x + \sqrt{9x^2-1}} dx$$

Bài giải

$$\text{a) } I = \int_{-1}^1 \frac{dx}{1+x+\sqrt{1+x^2}}$$

$$\text{Ta có: } I = \int_{-1}^1 \frac{1+x-\sqrt{1+x^2}}{(1+x)^2 - (1+x^2)} dx = \int_{-1}^1 \frac{1+x-\sqrt{1+x^2}}{2x} dx = \frac{1}{2} \int_{-1}^1 \left(\frac{1}{x} + 1 \right) dx - \int_{-1}^1 \frac{\sqrt{1+x^2}}{2x} dx$$

$$+ I_1 = \frac{1}{2} \int_{-1}^1 \left(\frac{1}{x} + 1 \right) dx = \frac{1}{2} [\ln|x| + x]_{-1}^1 = 1$$

$$+ I_2 = \int_{-1}^1 \frac{\sqrt{1+x^2}}{2x} dx. \text{ Đặt } t = \sqrt{1+x^2} \Rightarrow t^2 = 1+x^2 \Rightarrow 2t dt = 2x dx \Rightarrow I_2 = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \frac{t^2 dt}{2(t^2-1)} = 0$$

Vậy: $I = 1$.

Cách 2: Đặt $t = x + \sqrt{x^2 + 1} \Leftrightarrow t - x = \sqrt{x^2 + 1} \Rightarrow (t - x)^2 = x^2 + 1 \Leftrightarrow t^2 - 2tx = 1 \Leftrightarrow x = \frac{t^2 - 1}{2t}$

$$\Rightarrow dx = \left(\frac{1}{2} + \frac{1}{2t^2} \right) dt$$

Đổi cận: $x = -1 \Rightarrow t = -1 + \sqrt{2}; x = 1 \Rightarrow t = 1 + \sqrt{2}$

$$\begin{aligned} \Rightarrow I &= \int_{-1+\sqrt{2}}^{1+\sqrt{2}} \frac{(t^2+1)dx}{2t^2(1+t)} = \frac{1}{2} \int_{-1+\sqrt{2}}^{1+\sqrt{2}} \left(\frac{2}{t+1} + \frac{1}{t^2} - \frac{1}{t} \right) dt \\ &= \frac{1}{2} \left(2 \ln|t+1| - \frac{1}{t} - \ln|t| \right) \Big|_{-1+\sqrt{2}}^{1+\sqrt{2}} = \frac{1}{2} \left(\ln \frac{(t+1)^2}{t} - \frac{1}{t} \right) \Big|_{-1+\sqrt{2}}^{1+\sqrt{2}} \\ &= \frac{1}{2} (\ln(2+2\sqrt{2}) + 1 - \sqrt{2}) - \frac{1}{2} (\ln(2+2\sqrt{2}) - 1 - \sqrt{2}) = 1 \end{aligned}$$

$$\text{b) } I = \int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$$

Ta có: $I = \int_{\frac{1}{3}}^1 \left(\frac{1}{x^2} - 1 \right)^{\frac{1}{3}} \cdot \frac{1}{x^3} dx$

Đặt $t = \frac{1}{x^2} - 1 \Rightarrow dt = -\frac{2}{x^3} dx \Leftrightarrow \frac{dx}{x^3} = -\frac{dt}{2}$

Đổi cận: $x = \frac{1}{3} \Rightarrow t = 8; x = 1 \Rightarrow t = 0$

$$\Rightarrow I = -\frac{1}{2} \int_8^0 t^{\frac{1}{3}} dt = \frac{1}{2} \int_0^8 t^{\frac{1}{3}} dt = \frac{1}{2} \cdot \frac{3}{4} t^{\frac{4}{3}} \Big|_0^8 = 6$$

$$\text{c) } I = \int_0^1 \frac{1}{\sqrt{x^2 + x + 1}} dx = \int_0^1 \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}}$$

$$\text{Đặt: } \Rightarrow dt = \left(1 + \frac{2x+1}{2\sqrt{x^2+x+1}}\right) dx \Leftrightarrow dt = \frac{\sqrt{x^2+x+1} + x + \frac{1}{2}}{\sqrt{x^2+x+1}} dx \Rightarrow \frac{dx}{\sqrt{x^2+x+1}} = \frac{dt}{t}$$

$$\text{Đổi cận: } x=0 \Rightarrow t = \frac{3}{2}; x=1 \Rightarrow t = \frac{3}{2} + \sqrt{3} \quad t = x + \frac{1}{2} + \sqrt{x^2+x+1}$$

$$I = \int_{\frac{3}{2}}^{\frac{3}{2}+\sqrt{3}} \frac{dt}{t} = \ln|t| \Big|_{\frac{3}{2}}^{\frac{3}{2}+\sqrt{3}} = \ln\left(\frac{3}{2} + \sqrt{3}\right) - \ln\frac{3}{2} = \ln\frac{3+2\sqrt{3}}{3}$$

$$d) I = \int_0^3 \frac{x^2}{(1+\sqrt{1+x})^2(2+\sqrt{1+x})^2} dx$$

$$\text{Đặt } 2 + \sqrt{1+x} = t \Rightarrow t-2 = \sqrt{1+x} \Rightarrow (t-2)^2 = 1+x \Rightarrow 2(t-2)dt = dx$$

$$\text{Đổi cận: } x=0 \Rightarrow t=3; x=3 \Rightarrow t=4$$

$$\begin{aligned} \Rightarrow I &= \int_3^4 \frac{(t-2)^2 - 1}{(t-1)^2 t^2} \cdot 2(t-2)dt = \int_3^4 \frac{(t-1)^2(t-3)^2 \cdot 2(t-2)dt}{(t-1)^2 t^2} = \int_3^4 \frac{2(t-3)^2(t-2)dt}{t^2} \\ &= \int_3^4 \left(2t - 16 + \frac{42}{t} - \frac{36}{t^2}\right) dt = \left(t^2 - 16t + 42 \ln|t| + \frac{36}{t}\right) \Big|_3^4 = -12 + 42 \ln \frac{4}{3} \end{aligned}$$

$$e) I = \int_0^3 \frac{x^2}{2(x+1) + 2\sqrt{x+1} + x\sqrt{x+1}} dx$$

$$\text{Đặt: } t = \sqrt{x+1} \Rightarrow t^2 = x+1 \Rightarrow 2tdt = dx$$

$$\text{Đổi cận: } x=0 \Rightarrow t=1; x=3 \Rightarrow t=2$$

$$\Rightarrow I = \int_1^2 \frac{2t(t^2-1)^2 dt}{t(t+1)^2} = 2 \int_1^2 (t-1)^2 dt = \frac{2}{3} (t-1)^3 \Big|_1^2 = \frac{2}{3}$$

$$f) I = \int_1^{2\sqrt{2}} \frac{\sqrt[3]{x-x^3} + 2011x}{x^4} dx$$

$$\text{Ta có: } I = \int_1^{2\sqrt{2}} \frac{\sqrt[3]{\frac{1}{x^2} - 1}}{x^3} dx + \int_1^{2\sqrt{2}} \frac{2011}{x^3} dx = M + N$$

$$+) M = \int_1^{2\sqrt{2}} \frac{\sqrt[3]{\frac{1}{x^2} - 1}}{x^3} dx.$$

$$\text{Đặt } t = \sqrt[3]{\frac{1}{x^2} - 1} \Rightarrow t^3 = \frac{1}{x^2} - 1 \Rightarrow 3t^2 dt = -\frac{2}{x^3} dx \Rightarrow \frac{dx}{x^3} = -\frac{3}{2} t^2 dt$$

$$\text{Đổi cận: } x = 1 \Rightarrow t = 0; x = 2\sqrt{2} \Rightarrow t = -\frac{\sqrt[3]{7}}{2}$$

$$\Rightarrow M = -\frac{3}{2} \int_0^{-\frac{\sqrt[3]{7}}{2}} t^2 dt = -\frac{21\sqrt[3]{7}}{128}$$

$$+) N = \int_1^{2\sqrt{2}} \frac{2011}{x^3} dx = \int_1^{2\sqrt{2}} 2011x^{-3} dx = \left[-\frac{2011}{2x^2} \right]_1^{2\sqrt{2}} = \frac{14077}{16}$$

$$\Rightarrow I = \frac{14077}{16} - \frac{21\sqrt[3]{7}}{128}.$$

$$g) I = \int_{\frac{1}{\sqrt{3}}}^{2\sqrt{2}} \frac{x^4}{\left(x - \frac{1}{x}\right)\sqrt{x^2 + 1}} dx = \int_{\frac{1}{\sqrt{3}}}^{2\sqrt{2}} \frac{x^4 \cdot x dx}{(x^2 - 1)\sqrt{x^2 + 1}}$$

$$\text{Đặt } t = \sqrt{x^2 + 1} \Rightarrow dt = \frac{x dx}{\sqrt{x^2 + 1}}$$

$$\text{Đổi cận: } x = \frac{1}{\sqrt{3}} \Rightarrow t = 2; x = 2\sqrt{2} \Rightarrow t = 3$$

$$\Rightarrow I = \int_2^3 \frac{(t^2 - 1)^2}{t^2 - 2} dt = \int_2^3 \frac{t^4 - 2t^2 + 1}{t^2 - 2} dt = \int_2^3 t^2 dt + \int_2^3 \frac{1}{t^2 - 2} dt = \frac{19}{3} + \frac{\sqrt{2}}{4} \ln \left(\frac{4 + \sqrt{2}}{4 - \sqrt{2}} \right)$$

$$h) I = \int_{\frac{1}{3}}^{\frac{2}{3}} \frac{x}{3x + \sqrt{9x^2 - 1}} dx = \int_{\frac{1}{3}}^{\frac{2}{3}} x(3x - \sqrt{9x^2 - 1}) dx = \int_{\frac{1}{3}}^{\frac{2}{3}} 3x^2 dx - \int_{\frac{1}{3}}^{\frac{2}{3}} x\sqrt{9x^2 - 1} dx$$

$$+ I_1 = \int_{\frac{1}{3}}^{\frac{2}{3}} 3x^2 dx = x^3 \Big|_{\frac{1}{3}}^{\frac{2}{3}} = \frac{8}{27} - \frac{1}{27} = \frac{7}{27}$$

$$+ I_2 = \int_{\frac{1}{3}}^{\frac{2}{3}} x\sqrt{9x^2 - 1} dx = \frac{1}{18} \int_{\frac{1}{3}}^{\frac{2}{3}} \sqrt{9x^2 - 1} d(9x^2 - 1) = \frac{1}{27} (9x^2 - 1)^{\frac{3}{2}} \Big|_{\frac{1}{3}}^{\frac{2}{3}} = \frac{\sqrt{3}}{9}$$

$$\Rightarrow I = \frac{7 - 3\sqrt{3}}{27}$$

HT 5. Tính các tích phân sau:

$$\text{a) } I = \int_0^2 x^2 \sqrt{4 - x^2} dx$$

$$\text{b) } I_2 = \int_{-1}^0 \sqrt{-x^2 - 2x} dx$$

$$\text{c) } I = \int_0^1 \sqrt{3 + 2x - x^2} dx$$

$$\text{d) } I = \int_0^1 \frac{x^2 dx}{\sqrt{4 - x^6}}$$

$$\text{e) } I = \int_0^{\frac{1}{2}} \sqrt{1 - 2x} \sqrt{1 - x^2} dx$$

$$\text{f) } I = \int_0^1 \frac{x^2 dx}{\sqrt{3 + 2x - x^2}}$$

Bài giải

$$\text{a) } I = \int_0^2 x^2 \sqrt{4 - x^2} dx$$

$$\text{Đặt: } x = 2 \sin t, \text{ Với } t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \geq 0$$

$$\Rightarrow dx = 2 \cos t dt$$

$$\text{Đổi cận: } x = 0 \Rightarrow t = 0; x = 2 \Rightarrow t = \frac{\pi}{2}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} 4 \sin^2 t \sqrt{4 - 4 \sin^2 t} \cdot 2 \cos t dt = 16 \int_0^{\frac{\pi}{2}} \sin^2 t \sqrt{1 - \sin^2 t} \cdot \cos t dt$$

$$= 16 \int_0^{\frac{\pi}{2}} \sin^2 t \cdot |\cos t| \cos t dt = 16 \int_0^{\frac{\pi}{2}} \sin^2 t \cdot \cos^2 t dt = 4 \int_0^{\frac{\pi}{2}} \sin^2 4t dt = 2 \int_0^{\frac{\pi}{2}} (1 - \cos 8t) dt$$

$$= 2 \left(t - \frac{\sin 8t}{8} \right) \Big|_0^{\frac{\pi}{2}} = \pi$$

$$\text{b) } I_2 = \int_{-1}^0 \sqrt{-x^2 - 2x} dx = \int_{-1}^0 \sqrt{1 - (x+1)^2} dx$$

$$\text{Đặt: } x + 1 = \sin t, \text{ Với } t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \geq 0$$

$$\Rightarrow dx = \cos t dt$$

$$\text{Đổi cận: } x = -1 \Rightarrow t = 0; x = 0 \Rightarrow t = \frac{\pi}{2}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t} \cdot \cos t dt = \int_0^{\frac{\pi}{2}} \cos^2 t dt = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2t) dt$$

$$= \frac{1}{2} \left(t + \frac{\sin 2t}{2} \right) \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

$$\text{c) } I = \int_0^1 \sqrt{3+2x-x^2} dx = \int_0^1 \sqrt{4-(x-2)^2} dx$$

$$\text{Đặt: } x-2 = 2 \sin t, \text{ Với } t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \geq 0$$

$$\Rightarrow dx = 2 \cos t dt$$

$$\text{Đổi cận: } x=0 \Rightarrow t = -\frac{\pi}{2}; x=1 \Rightarrow t = -\frac{\pi}{6}$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \sqrt{4-4 \sin^2 t} \cdot 2 \cos t dt = 4 \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \cos^2 t dt = 2 \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} (1 + \cos 2t) dt$$

$$= 2 \left(t + \frac{\sin 2t}{2} \right) \Big|_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} = -\frac{\pi}{12} - \frac{\sqrt{3}}{4} + \frac{\pi}{4} = \frac{\pi}{6} - \frac{\sqrt{3}}{4}$$

$$\text{d) } I = \int_0^1 \frac{x^2 dx}{\sqrt{4-x^6}}$$

$$\text{Đặt } t = x^3 \Rightarrow dt = 3x^2 dx$$

$$\text{Đổi cận: } x=0 \Rightarrow t=0; x=1 \Rightarrow t=1$$

$$\Rightarrow I = \frac{1}{3} \int_0^1 \frac{dt}{\sqrt{4-t^2}}$$

$$\text{Đặt: } t = 2 \sin u, u \in \left[0; \frac{\pi}{2}\right] \Rightarrow dt = 2 \cos u du$$

$$\text{Đổi cận: } t=0 \Rightarrow u=0; t=1 \Rightarrow u = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{1}{3} \int_0^{\frac{\pi}{6}} \frac{2 \cos u du}{\sqrt{4-4 \sin^2 u}} = \frac{1}{3} \int_0^{\frac{\pi}{6}} du = \frac{u}{3} \Big|_0^{\frac{\pi}{6}} = \frac{\pi}{18}$$

Chú ý: Các em học sinh có thể đặt trực tiếp: $x^3 = 2 \sin t$

$$\text{e) } I = \int_0^{\frac{1}{2}} \sqrt{1-2x} \sqrt{1-x^2} dx$$

$$\text{Đặt } x = \sin t, \text{ Với } t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \geq 0; \cos t > \sin t$$

$$\Rightarrow dx = \cos t dt$$

$$\text{Đổi cận: } x=0 \Rightarrow t=0; x=\frac{1}{2} \Rightarrow t = \frac{\pi}{6}$$

$$\begin{aligned} \Rightarrow I &= \int_0^{\frac{\pi}{6}} \sqrt{1-2\sin t} \sqrt{1-\sin^2 t} \cdot \cos t \cdot dt = \int_0^{\frac{\pi}{6}} \sqrt{1-2\sin t} \cos t \cos t \cdot dt = \int_0^{\frac{\pi}{6}} \sqrt{(\sin t - \cos t)^2} \cos t \cdot dt \\ &= \int_0^{\frac{\pi}{6}} (\cos t - \sin t) \cos t \cdot dt = \int_0^{\frac{\pi}{6}} (\cos^2 t - \sin t \cdot \cos t) \cdot dt = \frac{1}{2} \int_0^{\frac{\pi}{6}} (1 + \cos 2t - \sin 2t) \cdot dt = \frac{1}{2} \left(t + \frac{\sin 2t}{2} + \frac{\cos 2t}{2} \right) \Big|_0^{\frac{\pi}{6}} \\ &= \frac{\pi}{12} + \frac{\sqrt{3}}{8} - \frac{1}{8} \end{aligned}$$

$$\text{f) } I = \int_0^1 \frac{x^2 dx}{\sqrt{3+2x-x^2}}$$

$$\text{Ta có: } I = \int_0^1 \frac{x^2 dx}{\sqrt{2^2 - (x-1)^2}}$$

$$\text{Đặt } x-1 = 2 \sin t. \text{ Với } t \in \left[-\frac{\pi}{2}; \frac{\pi}{2} \right] \Rightarrow \cos t \geq 0$$

$$\Rightarrow dx = 2 \cos t dt$$

$$\text{Đổi cận: } x=0 \Rightarrow t = -\frac{\pi}{2}; x=1 \Rightarrow t = -\frac{\pi}{6}$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \frac{(1+2\sin t)^2 2 \cos t}{\sqrt{4-(2\sin t)^2}} dt = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} (1+4\sin t+4\sin^2 t) dt = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} (1+4\sin t+2-2\cos 2t) dt$$

$$= (3t - 4 \cos t - \frac{\sin 2t}{2}) \Big|_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} = \frac{\pi}{2} + \frac{3\sqrt{3}}{2} - 4$$

HT 6. Tính các tích phân sau:

$$\text{a) } I = \int_{-2}^2 (x^5 + x^2) \sqrt{4-x^2} dx$$

$$\text{b) } I = \int_1^2 \frac{(3 - \sqrt{4-x^2}) dx}{2x^4}$$

$$\text{c) } I = \int_0^2 \sqrt{\frac{2-x}{x+2}} dx$$

$$\text{d) } I = \int_0^1 \left(\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} - 2x \ln(1+x) \right) dx$$

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Bài giải

$$\text{a) } I = \int_{-2}^2 (x^5 + x^2) \sqrt{4-x^2} dx$$

$$= \int_{-2}^2 (x^5 + x^2) \sqrt{4-x^2} dx = \int_{-2}^2 x^5 \sqrt{4-x^2} dx + \int_{-2}^2 x^2 \sqrt{4-x^2} dx = A + B.$$

$$+ \text{Tính } A = \int_{-2}^2 x^5 \sqrt{4-x^2} dx = \int_{-2}^2 x^4 \sqrt{4-x^2} x dx.$$

$$\text{Đặt } t = \sqrt{4-x^2} \Rightarrow t^2 = 4-x^2 \Rightarrow x dx = -t dt$$

$$\text{Đổi cận: } x = -2 \Rightarrow t = 0; x = 2 \Rightarrow t = 0$$

$$\Rightarrow I = \int_0^0 (4-t^2)^2 \cdot t^2 \cdot dt = 0$$

$$+ \text{Tính } B = \int_{-2}^2 x^2 \sqrt{4-x^2} dx.$$

$$\text{Đặt: } x = 2 \sin t, \text{ Với } t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow \cos t \geq 0$$

$$\Rightarrow dx = 2 \cos t dt$$

$$\text{Đổi cận: } x = -2 \Rightarrow t = -\frac{\pi}{2}; x = 2 \Rightarrow t = \frac{\pi}{2}$$

$$\Rightarrow B = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \sin^2 t \sqrt{4-4 \sin^2 t} \cdot 2 \cos t dt = 16 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t \sqrt{1-\sin^2 t} \cdot \cos t dt$$

$$= 16 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t \cdot |\cos t| \cos t dt = 16 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t \cdot \cos^2 t dt = 4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 4t dt = 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos 8t) dt$$

$$= 2 \left(t - \frac{\sin 8t}{8} \right) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2\pi$$

$$\text{Vậy, } I = 2\pi$$

$$b) I = \int_1^2 \frac{(3 - \sqrt{4-x^2}) dx}{2x^4}$$

$$\text{Ta có: } I = \int_1^2 \frac{3}{2x^4} dx - \int_1^2 \frac{\sqrt{4-x^2}}{2x^4} dx.$$

$$+ \text{Tính } I_1 = \int_1^2 \frac{3}{2x^4} dx = \frac{3}{2} \int_1^2 x^{-4} dx = \frac{7}{16}.$$

$$+ \text{Tính } I_2 = \int_1^2 \frac{\sqrt{4-x^2}}{2x^4} dx.$$

$$\text{Đặt } x = 2 \sin t \Rightarrow dx = 2 \cos t dt.$$

Đổi cận: $x = 1 \Rightarrow t = \frac{\pi}{6}; x = 2 \Rightarrow t = \frac{\pi}{2}$

$$\Rightarrow I_2 = \frac{1}{8} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^2 t dt}{\sin^4 t} = \frac{1}{8} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t \left(\frac{1}{\sin^2 t} \right) dt = -\frac{1}{8} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t d(\cot t) = \frac{\sqrt{3}}{8}$$

Vậy: $I = \frac{1}{16} (7 - 2\sqrt{3})$.

c) $I = \int_0^2 \sqrt{\frac{2-x}{x+2}} dx$

Đặt $x = 2 \cos t \Rightarrow dx = -2 \sin t dt$

Đổi cận: $x = 0 \Rightarrow t = \frac{\pi}{2}; x = 2 \Rightarrow t = 0$

$$\Rightarrow I = - \int_{\frac{\pi}{2}}^0 \sqrt{\frac{2-2\cos t}{2+2\cos t}} 2 \sin t dt = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 \frac{t}{2}}{\cos^2 \frac{t}{2}}} 2 \sin t dt.$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin \frac{t}{2}}{\cos \frac{t}{2}} 4 \sin \frac{t}{2} \cdot \cos \frac{t}{2} dt = \int_0^{\frac{\pi}{2}} 2(1 - \cos t) dt$$

$$= 2(t - \sin t) \Big|_0^{\frac{\pi}{2}} = \pi - 2$$

d) $I = \int_0^1 \left(\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} - 2x \ln(1+x) \right) dx$

Tính $H = \int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$. Đặt $\sqrt{x} = \cos t; t \in \left[0; \frac{\pi}{2}\right] \Rightarrow H = 2 - \frac{\pi}{2}$

Tính: $K = \int_0^1 2x \ln(1+x) dx$. Đặt $\begin{cases} u = \ln(1+x) \\ dv = 2x dx \end{cases} \Rightarrow K = \frac{1}{2}$

Vậy: $I = \frac{3}{2} - \frac{\pi}{2}$

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PHẦN IV TÍCH PHÂN HÀM LƯỢNG GIÁC

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HT 1. Tính các tích phân sau:

a) $I = \int_0^{\pi} \cos^4 x dx$

b) $I_2 = \int_0^{\frac{\pi}{2}} (\sin^6 x + \cos^6 x) dx$

c) $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \cdot \sin 5x dx$

d) $I = \int_0^{\frac{\pi}{2}} \frac{4 \sin^3 x}{1 + \cos x} dx$

e) $I = \int_0^{\frac{\pi}{3}} \frac{\sqrt{2} \sin\left(x - \frac{\pi}{4}\right)}{\cos x} dx$

f) $I = \int_0^{\frac{\pi}{4}} \frac{dx}{1 + \cos 2x}$

Bài giải

a) $I = \int_0^{\pi} \cos^4 x dx = \int_0^{\pi} (\cos^2 x)^2 dx = \int_0^{\pi} \left(\frac{1 + \cos 2x}{2}\right)^2 dx = \frac{1}{4} \int_0^{\pi} (1 + 2 \cos 2x + \cos^2 2x) dx$

$$= \frac{1}{4} \int_0^{\pi} \left(\frac{3}{2} + 2 \cos 2x + \frac{\cos 4x}{2}\right) dx = \frac{1}{4} \left(\frac{3}{2}x + \sin 2x + \frac{\sin 4x}{8}\right) \Big|_0^{\pi} = \frac{3\pi}{8}$$

b) $I_2 = \int_0^{\frac{\pi}{2}} (\sin^6 x + \cos^6 x) dx = \int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) dx$

$$= \int_0^{\frac{\pi}{2}} ((\sin^2 x + \cos^2 x)^2 - 3 \sin^2 x \cos^2 x) dx = \int_0^{\frac{\pi}{2}} \left(1 - \frac{3}{4} \sin^2 2x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{5}{8} + \frac{3}{8} \cos 4x\right) dx = \left(\frac{5}{8}x + \frac{3}{32} \sin 4x\right) \Big|_0^{\frac{\pi}{2}} = \frac{5\pi}{16}$$

c) $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \cdot \sin 5x dx = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos 3x - \cos 7x) dx = \frac{1}{2} \left(\frac{\sin 3x}{3} - \frac{\sin 7x}{7}\right) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\frac{4}{21}$

d) $I = \int_0^{\frac{\pi}{2}} \frac{4 \sin^3 x}{1 + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{4(1 - \cos^2 x) \sin x}{1 + \cos x} dx = \int_0^{\frac{\pi}{2}} 4(1 - \cos x) d(1 - \cos x) = 2(1 - \cos x)^2 \Big|_0^{\frac{\pi}{2}} = 2$

$$\text{e) } I = \int_0^{\frac{\pi}{3}} \frac{\sqrt{2} \sin\left(x - \frac{\pi}{4}\right)}{\cos x} dx = \int_0^{\frac{\pi}{3}} \frac{\sin x - \cos x}{\cos x} dx = \int_0^{\frac{\pi}{3}} \left(\frac{\sin x}{\cos x} - 1 \right) dx$$

$$= \left(-\ln|\cos x| - x \right) \Big|_0^{\frac{\pi}{3}} = \ln 2 - \frac{\pi}{3}$$

$$\text{f) } I = \int_0^{\frac{\pi}{4}} \frac{dx}{1 + \cos 2x} = \int_0^{\frac{\pi}{4}} \frac{dx}{2 \cos^2 x} = \frac{1}{2} \tan x \Big|_0^{\frac{\pi}{4}} = \frac{1}{2}$$

HT 2. Tính các tích phân sau:

$$\text{a) } I = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx \quad \text{b) } I = \int_0^{\frac{\pi}{2}} (\cos^3 x - 1) \cos^2 x dx$$

$$\text{c) } I = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^6 x}$$

$$\text{d) } I = \int_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) dx$$

$$\text{e) } I = \int_0^{\frac{\pi}{2}} \cos 2x (\sin^4 x + \cos^4 x) dx$$

Bài giải

$$\text{a) } I = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2x) \cos 2x dx = \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 + 2 \cos 2x + \cos 4x) dx$$

$$= \frac{1}{4} \left(x + \sin 2x + \frac{1}{4} \sin 4x \right) \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{8}$$

$$\text{b) } I = \int_0^{\frac{\pi}{2}} (\cos^3 x - 1) \cos^2 x dx = \int_0^{\frac{\pi}{2}} (\cos^5 x - \cos^2 x) dx$$

$$A = \int_0^{\frac{\pi}{2}} \cos^5 x dx = \int_0^{\frac{\pi}{2}} (1 - \sin^2 x)^2 d(\sin x) = \frac{8}{15}$$

$$B = \int_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + \cos 2x) dx = \frac{\pi}{4}$$

$$\text{Vậy } I = \frac{8}{15} - \frac{\pi}{4}.$$

$$\text{c) } I = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^6 x} = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x \cdot \cos^2 x} = \int_0^{\frac{\pi}{4}} (1 + 2 \tan^2 x + \tan^4 x) d(\tan x) = \frac{28}{15}.$$

$$\text{d) } I = \int_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) dx.$$

$$\text{Ta có: } (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) = \frac{33}{64} + \frac{7}{16} \cos 4x + \frac{3}{64} \cos 8x$$

$$\Rightarrow I = \frac{33}{128} \pi.$$

$$\text{e) } I = \int_0^{\frac{\pi}{2}} \cos 2x (\sin^4 x + \cos^4 x) dx$$

$$I = \int_0^{\frac{\pi}{2}} \cos 2x \left(1 - \frac{1}{2} \sin^2 2x\right) dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2} \sin^2 2x\right) d(\sin 2x) = 0$$

HT 3. Tính các tích phân sau :

$$\text{a) } I = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\cot x - \tan x - 2 \tan 2x}{\sin 4x} dx$$

$$\text{b) } I = \int_0^{\frac{\pi}{6}} \frac{1}{2 \sin x - \sqrt{3}} dx$$

$$\text{c) } I = \int_{\frac{\pi}{3}}^{\pi} \frac{dx}{2 + \sqrt{3} \sin x - \cos x}$$

$$\text{d) } I = \int \frac{\cos^2 \left(x + \frac{\pi}{8}\right)}{\sin 2x + \cos 2x + \sqrt{2}} dx$$

$$\text{e) } I = \int \frac{8 \cos^2 x - \sin 2x - 3}{\sin x - \cos x} dx \quad \text{f) } I = \int_0^{2\pi} \sqrt{1 + \sin x} dx$$

Bài giải

$$\text{a) } I = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\cot x - \tan x - 2 \tan 2x}{\sin 4x} dx$$

$$\text{Ta có: } I = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{2 \cot 2x - 2 \tan 2x}{\sin 4x} dx = \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{2 \cot 4x}{\sin 4x} dx = 2 \int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \frac{\cos 4x}{\sin^2 4x} dx = -\frac{1}{2 \sin 4x} \Big|_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \frac{2\sqrt{3} - 3}{6}$$

$$\text{b) } I = \int_0^{\frac{\pi}{6}} \frac{1}{2 \sin x - \sqrt{3}} dx$$

$$\text{Ta có: } I = \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{1}{\sin x - \sin \frac{\pi}{3}} dx = \int_0^{\frac{\pi}{6}} \frac{\frac{1}{2}}{\sin x - \sin \frac{\pi}{3}} dx$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos \frac{\pi}{3}}{\sin x - \sin \frac{\pi}{3}} dx = \int_0^{\frac{\pi}{6}} \frac{\cos \left(\left(\frac{x}{2} + \frac{\pi}{6} \right) - \left(\frac{x}{2} - \frac{\pi}{6} \right) \right)}{2 \cos \left(\frac{x}{2} + \frac{\pi}{6} \right) \cdot \sin \left(\frac{x}{2} - \frac{\pi}{6} \right)} dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{\cos \left(\frac{x}{2} - \frac{\pi}{6} \right)}{\sin \left(\frac{x}{2} - \frac{\pi}{6} \right)} dx + \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{\sin \left(\frac{x}{2} + \frac{\pi}{6} \right)}{\cos \left(\frac{x}{2} + \frac{\pi}{6} \right)} dx = \ln \left| \sin \left(\frac{x}{2} - \frac{\pi}{6} \right) \right| \Big|_0^{\frac{\pi}{6}} - \ln \left| \cos \left(\frac{x}{2} + \frac{\pi}{6} \right) \right| \Big|_0^{\frac{\pi}{6}} = \dots$$

$$\text{c) } I = \int_{\frac{\pi}{3}}^{\pi} \frac{dx}{2 + \sqrt{3} \sin x - \cos x}$$

$$I = \frac{1}{2} \int_{\frac{\pi}{3}}^{\pi} \frac{dx}{1 - \cos \left(x + \frac{\pi}{3} \right)} = I = \frac{1}{4} \int_{\frac{\pi}{3}}^{\pi} \frac{dx}{2 \sin^2 \left(\frac{x}{2} + \frac{\pi}{6} \right)} = \frac{1}{4\sqrt{3}}$$

$$\text{d) } I = \int \frac{\cos^2 \left(x + \frac{\pi}{8} \right)}{\sin 2x + \cos 2x + \sqrt{2}} dx$$

$$\text{Ta có: } I = \frac{1}{2\sqrt{2}} \int \frac{1 + \cos \left(2x + \frac{\pi}{4} \right)}{1 + \sin \left(2x + \frac{\pi}{4} \right)} dx$$

$$= \frac{1}{2\sqrt{2}} \left(\int \frac{\cos \left(2x + \frac{\pi}{4} \right)}{1 + \sin \left(2x + \frac{\pi}{4} \right)} dx + \int \frac{dx}{\left[\sin \left(x + \frac{\pi}{8} \right) + \cos \left(x + \frac{\pi}{8} \right) \right]^2} \right)$$

$$= \frac{1}{2\sqrt{2}} \left(\int \frac{\cos \left(2x + \frac{\pi}{4} \right)}{1 + \sin \left(2x + \frac{\pi}{4} \right)} dx + \frac{1}{2} \int \frac{dx}{\sin^2 \left(x + \frac{3\pi}{8} \right)} \right)$$

$$= \frac{1}{4\sqrt{2}} \left(\ln \left| 1 + \sin \left(2x + \frac{\pi}{4} \right) \right| - \cot \left(x + \frac{3\pi}{8} \right) \right) + C$$

$$\text{e) } I = \int \frac{8 \cos^2 x - \sin 2x - 3}{\sin x - \cos x} dx$$

$$I = \int \frac{(\sin x - \cos x)^2 + 4 \cos 2x}{\sin x - \cos x} dx = \int [(\sin x - \cos x - 4(\sin x + \cos x))] dx$$

$$= 3 \cos x - 5 \sin x + C.$$

$$\text{f) } I = \int_0^{2\pi} \sqrt{1 + \sin x} dx$$

$$\begin{aligned} I &= \int_0^{2\pi} \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2} dx = \int_0^{2\pi} \left| \sin \frac{x}{2} + \cos \frac{x}{2} \right| dx = \sqrt{2} \int_0^{2\pi} \left| \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| dx \\ &= \sqrt{2} \left[\int_0^{\frac{3\pi}{2}} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx - \int_{\frac{3\pi}{2}}^{2\pi} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx \right] = 4\sqrt{2} \end{aligned}$$

HT 4. Tính các tích phân sau:

$$1. I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(2 + \sin x)^2} dx$$

$$2. I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^6 x + \cos^6 x}} dx$$

$$3. I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x \sqrt{3 + \sin^2 x}} dx$$

$$4. I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x + \sin x) \sin x}{\sin^3 x + \sin^2 x} dx$$

$$5. I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{\cos^2 x + 4 \sin^2 x}} dx$$

$$6. I = \int_0^{\frac{\pi}{6}} \frac{\tan \left(x - \frac{\pi}{4} \right)}{\cos 2x} dx$$

$$7. I = 2 \int_1^2 \sqrt[6]{1 - \cos^3 x} \cdot \sin x \cdot \cos^5 x dx$$

$$8. I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos x \sqrt{1 + \cos^2 x}}$$

$$9. I = \int_0^{\frac{\pi}{2}} \frac{\cos 2x}{(\cos x - \sin x + 3)^3} dx$$

$$10. I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\cos^2 x \sqrt{\tan^4 x + 1}} dx$$

$$11. I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{1 + \cos^2 x} dx$$

$$12. I = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos 2x} dx$$

$$13. I = \int_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\sqrt{3 - \sin 2x}} dx$$

$$14. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x \cdot \sin \left(x + \frac{\pi}{4} \right)} dx$$

$$15. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x \cdot \cos^4 x}$$

Bài giải

$$1. I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(2 + \sin x)^2} dx$$

Ta có: $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(2 + \sin x)^2} dx = 2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{(2 + \sin x)^2} dx$. Đặt $t = 2 + \sin x$.

$$\Rightarrow I = 2 \int_2^3 \frac{t-2}{t^2} dt = 2 \int_2^3 \left(\frac{1}{t} - \frac{2}{t^2} \right) dt = 2 \left(\ln t + \frac{2}{t} \right) \Big|_2^3 = 2 \ln \frac{3}{2} - \frac{2}{3}$$

2. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^6 x + \cos^6 x}} dx$

• $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{1 - \frac{3}{4} \sin^2 2x}} dx$. Đặt $t = 1 - \frac{3}{4} \sin^2 2x \Rightarrow I = \int_1^{\frac{1}{4}} \left(-\frac{2}{3} \frac{1}{\sqrt{t}} \right) dt = \frac{4}{3} \sqrt{t} \Big|_{\frac{1}{4}}^1 = \frac{2}{3}$.

3. $I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x \sqrt{3 + \sin^2 x}} dx$

Đặt $t = \sqrt{3 + \sin^2 x} = \sqrt{4 - \cos^2 x}$. Ta có: $\cos^2 x = 4 - t^2$ và $dt = \frac{\sin x \cos x}{\sqrt{3 + \sin^2 x}} dx$.

$$I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x \sqrt{3 + \sin^2 x}} dx = \int_0^{\frac{\pi}{3}} \frac{\sin x \cos x}{\cos^2 x \sqrt{3 + \sin^2 x}} dx = \int_{\frac{\sqrt{15}}{3}}^{\frac{\sqrt{15}}{2}} \frac{dt}{\sqrt{3} (4 - t^2)} = \frac{1}{4} \int_{\frac{\sqrt{15}}{3}}^{\frac{\sqrt{15}}{2}} \left(\frac{1}{t+2} - \frac{1}{t-2} \right) dt$$

$$= \frac{1}{4} \ln \left| \frac{t+2}{t-2} \right| \Big|_{\frac{\sqrt{15}}{3}}^{\frac{\sqrt{15}}{2}} = \frac{1}{4} \left(\ln \left| \frac{\sqrt{15}+4}{\sqrt{15}-4} \right| - \ln \left| \frac{\sqrt{3}+2}{\sqrt{3}-2} \right| \right) = \frac{1}{2} (\ln(\sqrt{15}+4) - \ln(\sqrt{3}+2)).$$

4. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x + \sin x) \sin x}{\sin^3 x + \sin^2 x} dx$

$$I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1 + \sin x}$$

+ Tính $I_1 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx$. Đặt $\begin{cases} u = x \\ dv = \frac{dx}{\sin^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x \end{cases} \Rightarrow I_1 = \frac{\pi}{\sqrt{3}}$

+ Tính $I_2 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1 + \sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1 + \cos\left(\frac{\pi}{2} - x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2 \cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)} = 4 - 2\sqrt{3}$

Vậy: $I = \frac{\pi}{\sqrt{3}} + 4 - 2\sqrt{3}$.

$$5. I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{\cos^2 x + 4 \sin^2 x}} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{2 \sin x \cos x}{\sqrt{3 \sin^2 x + 1}} dx. \text{ Đặt } u = \sqrt{3 \sin^2 x + 1} \Rightarrow I = \int_1^2 \frac{\frac{2}{3} u du}{u} = \frac{2}{3} \int_1^2 du = \frac{2}{3}$$

$$6. I = \int_0^{\frac{\pi}{6}} \frac{\tan\left(x - \frac{\pi}{4}\right)}{\cos 2x} dx$$

$$I = \int_0^{\frac{\pi}{6}} \frac{\tan\left(x - \frac{\pi}{4}\right)}{\cos 2x} dx = - \int_0^{\frac{\pi}{6}} \frac{\tan^2 x + 1}{(\tan x + 1)^2} dx. \text{ Đặt } t = \tan x \Rightarrow dt = \frac{1}{\cos^2 x} dx = (\tan^2 x + 1) dx$$

$$\Rightarrow I = - \int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(t+1)^2} = \frac{1}{t+1} \Big|_0^{\frac{1}{\sqrt{3}}} = \frac{1 - \sqrt{3}}{2}.$$

$$7. I = 2 \int_1^2 \sqrt[6]{1 - \cos^3 x} \cdot \sin x \cdot \cos^5 x dx$$

$$\text{Đặt } t = \sqrt[6]{1 - \cos^3 x} \Leftrightarrow t^6 = 1 - \cos^3 x \Rightarrow 6t^5 dt = 3 \cos^2 x \sin x dx \Rightarrow dx = \frac{2t^5 dt}{\cos^2 x \sin x}$$

$$\Rightarrow I = 2 \int_0^1 t^6 (1 - t^6) dt = 2 \left(\frac{t^7}{7} - \frac{t^{13}}{13} \right) \Big|_0^1 = \frac{12}{91}$$

$$8. I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos x \sqrt{1 + \cos^2 x}}$$

$$\text{Ta có: } I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos^2 x \sqrt{\tan^2 x + 2}}. \text{ Đặt } t = \sqrt{2 + \tan^2 x} \Rightarrow t^2 = 2 + \tan^2 x \Rightarrow t dt = \frac{\tan x}{\cos^2 x} dx$$

$$\Rightarrow I = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} \frac{t dt}{t} = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} dt = \sqrt{3} - \sqrt{2}$$

$$9. I = \int_0^{\frac{\pi}{2}} \frac{\cos 2x}{(\cos x - \sin x + 3)^3} dx$$

$$\text{Đặt } t = \cos x - \sin x + 3 \Rightarrow I = \int_2^4 \frac{t-3}{t^3} dt = -\frac{1}{32}.$$

$$10. I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\cos^2 x \sqrt{\tan^4 x + 1}} dx$$

Ta có: $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^4 x + \cos^4 x}} dx$. Đặt $t = \sqrt{\sin^4 x + \cos^4 x} \Rightarrow I = -2 \int_1^{\frac{\sqrt{2}}{2}} dt = 2 - \sqrt{2}$.

$$11. I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{1 + \cos^2 x} dx$$

Ta có: $I = \int_0^{\frac{\pi}{4}} \frac{2 \sin 2x (2 \cos^2 x - 1)}{1 + \cos^2 x} dx$. Đặt $t = \cos^2 x \Rightarrow I = - \int_1^{\frac{1}{2}} \frac{2(2t-1)}{t+1} dt = 2 - 6 \ln \frac{1}{3}$.

$$12. I = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos 2x} dx$$

Ta có: $I = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos^2 x - \sin^2 x} dx = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos^2 x (1 - \tan^2 x)} dx$.

Đặt $t = \tan x \Rightarrow I = \int_0^{\frac{\sqrt{3}}{3}} \frac{t^3}{1-t^2} dt = \frac{1}{6} - \frac{1}{2} \ln \frac{2}{3}$.

$$13. I = \int_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\sqrt{3 - \sin 2x}} dx$$

Đặt $u = \sin x + \cos x \Rightarrow I = \int_1^{\sqrt{2}} \frac{du}{\sqrt{4-u^2}}$.

Đặt $u = 2 \sin t \Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{2 \cos t dt}{\sqrt{4-4 \sin^2 t}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} dt = \frac{\pi}{12}$.

$$14. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x \cdot \sin \left(x + \frac{\pi}{4}\right)} dx$$

$I = \sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin^2 x (1 + \cot x)} dx$. Đặt $1 + \cot x = t \Rightarrow \frac{1}{\sin^2 x} dx = -dt$

$$\Rightarrow I = \sqrt{2} \int_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} \frac{t-1}{t} dt = \sqrt{2} (t - \ln t) \Big|_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} = \sqrt{2} \left(\frac{2}{\sqrt{3}} - \ln \sqrt{3} \right)$$

$$15. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x \cdot \cos^4 x}$$

Ta có: $I = 4 \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 2x \cdot \cos^2 x}$. Đặt $t = \tan x \Rightarrow dx = \frac{dt}{1+t^2}$

$$\Rightarrow I = \int_1^{\sqrt{3}} \frac{\sqrt{3}(1+t^2)^2 dt}{t^2} = \int_1^{\sqrt{3}} \left(\frac{1}{t} + 2 + t^2 \right) dt = \left(-\frac{1}{t} + 2t + \frac{t^3}{3} \right) \Big|_1^{\sqrt{3}} = \frac{8\sqrt{3}-4}{3}$$

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HT 5. Tính các tích phân sau:

$$1. I = \int \frac{\sin 2x dx}{3 + 4 \sin x - \cos 2x}$$

$$2. I = \int \frac{dx}{\sin^3 x \cdot \cos^5 x}$$

$$3. I = \int \frac{dx}{\sin x \cdot \cos^3 x}$$

$$4. I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x \cdot \cos x}{1 + \cos x} dx$$

$$5. I = \int_0^{\frac{\pi}{3}} \sin^2 x \tan x dx$$

$$6. I = \int \sin^2 x (2 - \sqrt{1 + \cos 2x}) dx$$

$$7. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x \cdot \cos^4 x}$$

$$8. I = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x} dx$$

$$11. I = \int_0^{\frac{\pi}{6}} \frac{1}{\sin x + \sqrt{3} \cos x} dx$$

$$12. I = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3} \sin 2x + 2 \cos^2 x} dx$$

$$13. I = \int_0^{\frac{\pi}{4}} \frac{\sin x}{5 \sin x \cdot \cos^2 x + 2 \cos x} dx$$

$$14. I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^2 x dx}{\cos^4 x (\tan^2 x - 2 \tan x + 5)}$$

$$15. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin 3x} dx$$

$$16. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx$$

$$17. \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sqrt[4]{\sin^3 x \cdot \cos^5 x}}$$

$$9. I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{(\sin x + \sqrt{3} \cos x)^3} dx$$

$$10. I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x \sqrt{1 - \cos^2 x}}{\cos^2 x} dx$$

$$18. I = \int_0^{\pi} x \left(\frac{\cos^3 x + \cos x + \sin x}{1 + \cos^2 x} \right) dx$$

$$19. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x \sqrt{3 + \cos^2 x}} dx$$

Bài giải

$$1. I = \int \frac{\sin 2x dx}{3 + 4 \sin x - \cos 2x}$$

Ta có: $I = \int \frac{2 \sin x \cos x}{2 \sin^2 x + 4 \sin x + 2} dx$. Đặt $t = \sin x \Rightarrow I = \ln|\sin x + 1| + \frac{1}{\sin x + 1} + C$

$$2. I = \int \frac{dx}{\sin^3 x \cdot \cos^5 x}$$

$$I = \int \frac{dx}{\sin^3 x \cdot \cos^3 x \cdot \cos^2 x} = 8 \int \frac{dx}{\sin^3 2x \cdot \cos^2 x}$$

Đặt $t = \tan x$. $I = \int \left(t^3 + 3t + \frac{3}{t} + t^{-3} \right) dt = \frac{1}{4} \tan^4 x + \frac{3}{2} \tan^2 x + 3 \ln|\tan x| - \frac{1}{2 \tan^2 x} + C$

Chú ý: $\sin 2x = \frac{2t}{1+t^2}$.

$$3. I = \int \frac{dx}{\sin x \cdot \cos^3 x}$$

$$I = \int \frac{dx}{\sin x \cdot \cos x \cdot \cos^2 x} = 2 \int \frac{dx}{\sin 2x \cdot \cos^2 x}$$
. Đặt $t = \tan x \Rightarrow dt = \frac{dx}{\cos^2 x}$; $\sin 2x = \frac{2t}{1+t^2}$

$$\Rightarrow I = 2 \int \frac{dt}{\frac{2t}{1+t^2}} = \int \frac{t^2 + 1}{t} dt = \int \left(t + \frac{1}{t} \right) dt = \frac{t^2}{2} + \ln|t| + C = \frac{\tan^2 x}{2} + \ln|\tan x| + C$$

$$4. I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x \cdot \cos x}{1 + \cos x} dx$$

Ta có: $I = 2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos^2 x}{1 + \cos x} dx$. Đặt $t = 1 + \cos x \Rightarrow I = 2 \int_1^2 \frac{(t-1)^2}{t} dt = 2 \ln 2 - 1$

$$5. I = \int_0^{\frac{\pi}{3}} \sin^2 x \tan x dx$$

$$\text{Ta có: } I = \int_0^{\frac{\pi}{3}} \sin^2 x \cdot \frac{\sin x}{\cos x} dx = \int_0^{\frac{\pi}{3}} \frac{(1 - \cos^2 x) \sin x}{\cos x} dx. \text{ Đặt } t = \cos x$$

$$\Rightarrow I = - \int_1^{\frac{1}{2}} \frac{1 - u^2}{u} du = \ln 2 - \frac{3}{8}$$

$$6. I = \int_{\frac{\pi}{2}}^{\pi} \sin^2 x (2 - \sqrt{1 + \cos 2x}) dx$$

$$\text{Ta có: } I = \int_{\frac{\pi}{2}}^{\pi} 2 \sin^2 x dx - \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \sqrt{1 + \cos 2x} dx = H + K$$

$$+ H = \int_{\frac{\pi}{2}}^{\pi} 2 \sin^2 x dx = \int_{\frac{\pi}{2}}^{\pi} (1 - \cos 2x) dx = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$+ K = \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \sqrt{2 \cos^2 x} dx = -\sqrt{2} \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \cos x dx = -\sqrt{2} \int_{\frac{\pi}{2}}^{\pi} \sin^2 x d(\sin x) = \frac{\sqrt{2}}{3}$$

$$\Rightarrow I = \frac{\pi}{2} - \frac{\sqrt{2}}{3}$$

$$7. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x \cdot \cos^4 x}$$

$$I = 4 \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 2x \cdot \cos^2 x}. \text{ Đặt } t = \tan x \Rightarrow dt = \frac{dx}{\cos^2 x}.$$

$$I = \int_1^{\sqrt{3}} \frac{(1+t^2)^2 dt}{t^2} = \int_1^{\sqrt{3}} \left(\frac{1}{t^2} + 2 + t^2 \right) dt = \left(-\frac{1}{t} + 2t + \frac{t^3}{3} \right) \Big|_1^{\sqrt{3}} = \frac{8\sqrt{3} - 4}{3}$$

$$8. I = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x} dx$$

$$I = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x} dx = \int_0^{\frac{\pi}{6}} \frac{\sin x}{2 \cos^2 x - 1} dx. \text{ Đặt } t = \cos x \Rightarrow dt = -\sin x dx$$

Đổi cận: $x = 0 \Rightarrow t = 1$; $x = \frac{\pi}{6} \Rightarrow t = \frac{\sqrt{3}}{2}$

$$\text{Ta được } I = -\int_1^{\frac{\sqrt{3}}{2}} \frac{1}{2t^2 - 1} dt = \frac{1}{2\sqrt{2}} \ln \left| \frac{2t - \sqrt{2}}{2t + \sqrt{2}} \right| \Big|_{\frac{\sqrt{3}}{2}}^1 = \frac{1}{2\sqrt{2}} \ln \left| \frac{3 - 2\sqrt{2}}{5 - 2\sqrt{6}} \right|$$

$$9. I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{(\sin x + \sqrt{3} \cos x)^3} dx$$

Ta có: $\sin x + \sqrt{3} \cos x = 2 \cos \left(x - \frac{\pi}{6} \right)$;

$$\sin x = \sin \left(\left(x - \frac{\pi}{6} \right) + \frac{\pi}{6} \right) = \frac{\sqrt{3}}{2} \sin \left(x - \frac{\pi}{6} \right) + \frac{1}{2} \cos \left(x - \frac{\pi}{6} \right)$$

$$I = \frac{\sqrt{3}}{16} \int_0^{\frac{\pi}{2}} \frac{\sin \left(x - \frac{\pi}{6} \right) dx}{\cos^3 \left(x - \frac{\pi}{6} \right)} + \frac{1}{16} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2 \left(x - \frac{\pi}{6} \right)} = \frac{\sqrt{3}}{6}$$

$$10. I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x \sqrt{1 - \cos^2 x}}{\cos^2 x} dx$$

$$\begin{aligned} I &= \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x} \sqrt{1 - \cos^2 x} dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x} |\sin x| dx = \int_{-\frac{\pi}{3}}^0 \frac{\sin x}{\cos^2 x} |\sin x| dx + \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x} |\sin x| dx \\ &= -\int_{-\frac{\pi}{3}}^0 \frac{\sin^2 x}{\cos^2 x} dx + \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x} dx = \frac{7\pi}{12} - \sqrt{3} - 1. \end{aligned}$$

$$11. I = \int_0^{\frac{\pi}{6}} \frac{1}{\sin x + \sqrt{3} \cos x} dx$$

$$I = \int_0^{\frac{\pi}{6}} \frac{1}{\sin x + \sqrt{3} \cos x} dx = \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{1}{\sin \left(x + \frac{\pi}{3} \right)} dx = \frac{1}{2} \int_0^{\frac{\pi}{6}} \frac{\sin \left(x + \frac{\pi}{3} \right)}{1 - \cos^2 \left(x + \frac{\pi}{3} \right)} dx.$$

$$\text{Đặt } t = \cos \left(x + \frac{\pi}{3} \right) \Rightarrow dt = -\sin \left(x + \frac{\pi}{3} \right) dx \Rightarrow I = \frac{1}{2} \int_0^{\frac{1}{2}} \frac{1}{1 - t^2} dt = \frac{1}{4} \ln 3$$

$$12. I = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3} \sin 2x + 2 \cos^2 x} dx$$

$$I = \int_0^{\frac{\pi}{2}} |\sin x - \sqrt{3} \cos x| dx = I = \int_0^{\frac{\pi}{3}} |\sin x - \sqrt{3} \cos x| dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} |\sin x - \sqrt{3} \cos x| dx = 3 - \sqrt{3}$$

$$13. I = \int_0^{\frac{\pi}{4}} \frac{\sin x}{5 \sin x \cdot \cos^2 x + 2 \cos x} dx$$

$$\text{Ta có: } I = \int_0^{\frac{\pi}{4}} \frac{\tan x}{5 \tan x + 2(1 + \tan^2 x)} \cdot \frac{1}{\cos^2 x} dx. \text{ Đặt } t = \tan x,$$

$$\Rightarrow I = \int_0^1 \frac{t}{2t^2 + 5t + 2} dt = \frac{1}{3} \int_0^1 \left(\frac{2}{t+2} - \frac{1}{2t+1} \right) dt = \frac{1}{2} \ln 3 - \frac{2}{3} \ln 2$$

$$14. I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^2 x dx}{\cos^4 x (\tan^2 x - 2 \tan x + 5)}$$

$$\text{Đặt } t = \tan x \Rightarrow dx = \frac{dt}{1+t^2} \Rightarrow I = \int_{-1}^1 \frac{t^2 dt}{t^2 - 2t + 5} = 2 + \ln \frac{2}{3} - 3 \int_{-1}^1 \frac{dt}{t^2 - 2t + 5}$$

$$\text{Tính } I_1 = \int_{-1}^1 \frac{dt}{t^2 - 2t + 5}. \text{ Đặt } \frac{t-1}{2} = \tan u \Rightarrow I_1 = \frac{1}{2} \int_{-\frac{\pi}{4}}^0 du = \frac{\pi}{8}. \text{ Vậy } I = 2 + \ln \frac{2}{3} - \frac{3\pi}{8}.$$

$$15. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin 3x} dx.$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{3 \sin x - 4 \sin^3 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin x}{4 \cos^2 x - 1} dx$$

$$\text{Đặt } t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow I = - \int_{\frac{\sqrt{3}}{2}}^0 \frac{dt}{4t^2 - 1} = \frac{1}{4} \int_0^{\frac{\sqrt{3}}{2}} \frac{dt}{t^2 - \frac{1}{4}} = \frac{1}{4} \ln(2 - \sqrt{3})$$

$$16. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} dx$$

Ta có: $\sqrt{1 + \sin 2x} = |\sin x + \cos x| = \sin x + \cos x$ (vì $x \in \left[\frac{\pi}{4}; \frac{\pi}{2}\right]$)

$$\Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sin x + \cos x} dx. \text{ Đặt } t = \sin x + \cos x \Rightarrow dt = (\cos x - \sin x) dx$$

$$\Rightarrow I = \int_1^{\sqrt{2}} \frac{1}{t} dt = \ln|t| \Big|_1^{\sqrt{2}} = \frac{1}{2} \ln 2$$

$$17. \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sqrt[4]{\sin^3 x \cdot \cos^5 x}}$$

Ta có: $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sqrt[4]{\frac{\sin^3 x}{\cos^3 x} \cdot \cos^8 x}} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sqrt[4]{\tan^3 x}} \cdot \frac{1}{\cos^2 x} dx.$

Đặt $t = \tan x \Rightarrow I = \int_1^{\sqrt{3}} t^{-\frac{3}{4}} dt = 4(\sqrt[4]{3} - 1)$

$$18. I = \int_0^{\pi} x \left(\frac{\cos^3 x + \cos x + \sin x}{1 + \cos^2 x} \right) dx$$

Ta có: $I = \int_0^{\pi} x \left(\frac{\cos x(1 + \cos^2 x) + \sin x}{1 + \cos^2 x} \right) dx = \int_0^{\pi} x \cdot \cos x \cdot dx + \int_0^{\pi} \frac{x \cdot \sin x}{1 + \cos^2 x} dx = J + K$

+ Tính $J = \int_0^{\pi} x \cdot \cos x \cdot dx$. Đặt $\begin{cases} u = x \\ dv = \cos x dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \sin x \end{cases} \Rightarrow J = -2$

+ Tính $K = \int_0^{\pi} \frac{x \cdot \sin x}{1 + \cos^2 x} dx$. Đặt $x = \pi - t \Rightarrow dx = -dt$

$$\Rightarrow K = \int_0^{\pi} \frac{(\pi - t) \cdot \sin(\pi - t)}{1 + \cos^2(\pi - t)} dt = \int_0^{\pi} \frac{(\pi - t) \cdot \sin t}{1 + \cos^2 t} dt = \int_0^{\pi} \frac{(\pi - x) \cdot \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow 2K = \int_0^{\pi} \frac{(x + \pi - x) \cdot \sin x}{1 + \cos^2 x} dx = \pi \int_0^{\pi} \frac{\sin x \cdot dx}{1 + \cos^2 x} \Rightarrow K = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x \cdot dx}{1 + \cos^2 x}$$

$$\text{Đặt } t = \cos x \Rightarrow K = \frac{\pi}{2} \int_{-1}^1 \frac{dt}{1+t^2}, \quad \text{đặt } t = \tan u \Rightarrow dt = (1 + \tan^2 u) du$$

$$\Rightarrow K = \frac{\pi}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1 + \tan^2 u) du}{1 + \tan^2 u} = \frac{\pi}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} du = \frac{\pi}{2} \cdot u \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi^2}{4}$$

$$\text{Vậy } I = \frac{\pi^2}{4} - 2$$

$$19. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x \sqrt{3 + \cos^2 x}} dx$$

$$\text{Ta có: } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^2 x \sqrt{3 + \cos^2 x}} dx. \text{ Đặt } t = \sqrt{3 + \cos^2 x}$$

$$\Rightarrow I = \int_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{15}}{2}} \frac{dt}{4-t^2} = \frac{1}{2} (\ln(\sqrt{15} + 4) - \ln(\sqrt{3} + 2))$$

HT 6. Tính các tích phân sau:

$$1. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot \sqrt{\sin^2 x + \frac{1}{2}} dx$$

$$2. I = \int_0^{\frac{\pi}{2}} \frac{3 \sin x + 4 \cos x}{3 \sin^2 x + 4 \cos^2 x} dx$$

$$3. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos x \sqrt{1 + \cos^2 x}} dx$$

$$4. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin \left(x + \frac{\pi}{4} \right)}{2 \sin x \cos x - 3} dx$$

Bài giải

$$1. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot \sqrt{\sin^2 x + \frac{1}{2}} dx$$

$$\bullet \text{Đặt } \cos x = \sqrt{\frac{3}{2}} \sin t, \left(0 \leq t \leq \frac{\pi}{2} \right) \Rightarrow I = \frac{3}{2} \int_0^{\frac{\pi}{4}} \cos^2 t dt = \frac{3}{2} \left(\frac{\pi}{4} + \frac{1}{2} \right).$$

$$2. I = \int_0^{\frac{\pi}{2}} \frac{3 \sin x + 4 \cos x}{3 \sin^2 x + 4 \cos^2 x} dx$$

$$\bullet I = \int_0^{\frac{\pi}{2}} \frac{3 \sin x + 4 \cos x}{3 + \cos^2 x} dx = \int_0^{\frac{\pi}{2}} \frac{3 \sin x}{3 + \cos^2 x} dx + \int_0^{\frac{\pi}{2}} \frac{4 \cos x}{3 + \cos^2 x} dx = \int_0^{\frac{\pi}{2}} \frac{3 \sin x}{3 + \cos^2 x} dx + \int_0^{\frac{\pi}{2}} \frac{4 \cos x}{4 - \sin^2 x} dx$$

$$+ \text{Tính } I_1 = \int_0^{\frac{\pi}{2}} \frac{3 \sin x}{3 + \cos^2 x} dx. \text{ Đặt } t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow I_1 = \int_0^1 \frac{3 dt}{3 + t^2}$$

$$\text{Đặt } t = \sqrt{3} \tan u \Rightarrow dt = \sqrt{3}(1 + \tan^2 u) du \Rightarrow I_1 = \int_0^{\frac{\pi}{6}} \frac{3\sqrt{3}(1 + \tan^2 u) du}{3(1 + \tan^2 u)} = \frac{\pi\sqrt{3}}{6}$$

$$+ \text{Tính } I_2 = \int_0^{\frac{\pi}{2}} \frac{4 \cos x}{4 - \sin^2 x} dx. \text{ Đặt } t_1 = \sin x \Rightarrow dt_1 = \cos x dx \Rightarrow I_2 = \int_0^1 \frac{4 dt_1}{4 - t_1^2} = \ln 3$$

$$\text{Vậy: } I = \frac{\pi\sqrt{3}}{6} + \ln 3$$

$$3. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos x \sqrt{1 + \cos^2 x}} dx$$

$$\bullet \text{Ta có: } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos^2 x \sqrt{\frac{1}{\cos^2 x} + 1}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos^2 x \sqrt{\tan^2 x + 2}} dx$$

$$\text{Đặt } u = \tan x \Rightarrow du = \frac{1}{\cos^2 x} dx \Rightarrow I = \int_{\frac{1}{\sqrt{3}}}^1 \frac{u}{\sqrt{u^2 + 2}} dx. \text{ Đặt } t = \sqrt{u^2 + 2} \Rightarrow dt = \frac{u}{\sqrt{u^2 + 2}} du.$$

$$\Rightarrow I = \int_{\frac{\sqrt{7}}{\sqrt{3}}}^{\sqrt{3}} dt = t \Big|_{\frac{\sqrt{7}}{\sqrt{3}}}^{\sqrt{3}} = \sqrt{3} - \frac{\sqrt{7}}{\sqrt{3}} = \frac{3 - \sqrt{7}}{\sqrt{3}}.$$

$$4. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin \left(x + \frac{\pi}{4} \right)}{2 \sin x \cos x - 3} dx$$

• Ta có: $I = -\frac{1}{\sqrt{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x + \cos x}{(\sin x - \cos x)^2 + 2} dx$. Đặt $t = \sin x - \cos x \Rightarrow I = -\frac{1}{\sqrt{2}} \int_0^1 \frac{1}{t^2 + 2} dt$

Đặt $t = \sqrt{2} \tan u \Rightarrow I = -\frac{1}{\sqrt{2}} \int_0^{\arctan \frac{1}{\sqrt{2}}} \frac{\sqrt{2}(1 + \tan^2 u)}{2 \tan^2 u + 2} du = -\frac{1}{2} \arctan \frac{1}{\sqrt{2}}$

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HT 7. Tính các tích phân sau:

1. $I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{x \sin x}{\cos^2 x} dx$

2. $I = \int_0^{\frac{\pi}{2}} \left(\frac{1 + \sin x}{1 + \cos x} \right) e^x dx$

3. $I = \int_0^{\frac{\pi}{4}} \frac{x \cos 2x}{(1 + \sin 2x)^2} dx$

Bài giải

1. $I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{x \sin x}{\cos^2 x} dx$.

• Sử dụng công thức tích phân từng phần ta có:

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x d\left(\frac{1}{\cos x}\right) = \frac{x}{\cos x} \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{\cos x} = \frac{4\pi}{3} - J, \text{ với } J = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{\cos x}$$

Để tính J ta đặt $t = \sin x$. Khi đó $J = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{\cos x} = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{dt}{\sqrt{1-t^2}} = -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| \Big|_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} = -\ln \frac{2-\sqrt{3}}{2+\sqrt{3}}$

Vậy $I = \frac{4\pi}{3} - \ln \frac{2-\sqrt{3}}{2+\sqrt{3}}$.

2. $I = \int_0^{\frac{\pi}{2}} \left(\frac{1 + \sin x}{1 + \cos x} \right) e^x dx$

Ta có: $\frac{1 + \sin x}{1 + \cos x} = \frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \frac{1}{2 \cos^2 \frac{x}{2}} + \tan \frac{x}{2}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{e^x dx}{2 \cos^2 \frac{x}{2}} + \int_0^{\frac{\pi}{2}} e^x \tan \frac{x}{2} dx = e^2$$

$$3. I = \int_0^{\frac{\pi}{4}} \frac{x \cos 2x}{(1 + \sin 2x)^2} dx$$

$$\text{Đặt } \begin{cases} u = x \\ dv = \frac{\cos 2x}{(1 + \sin 2x)^2} dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\frac{1}{1 + \sin 2x} \end{cases}$$

$$\begin{aligned} \Rightarrow I &= x \cdot \left(-\frac{1}{2} \cdot \frac{1}{1 + \sin 2x} \right) \Big|_0^{\frac{\pi}{4}} + \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin 2x} dx = -\frac{\pi}{16} + \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{2}} \cdot \frac{1}{\cos^2 \left(x - \frac{\pi}{4} \right)} dx \\ &= -\frac{\pi}{16} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \tan \left(x - \frac{\pi}{4} \right) \Big|_0^{\frac{\pi}{4}} = -\frac{\pi}{16} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} (0 + 1) = \frac{\sqrt{2}}{4} - \frac{\pi}{16} \end{aligned}$$

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PHẦN V TÍCH PHÂN HÀM MŨ VÀ LOGARIT

<http://www.LuuHuyThuong.blogspot.com>

HT 1. Tính các tích phân sau:

1. $I = \int \frac{e^{2x}}{1 + \sqrt{e^x}} dx$

2. $I = \int \frac{(x^2 + x)e^x}{x + e^{-x}} dx$

3. $I = \int \frac{dx}{\sqrt{e^{2x} + 9}}$

4. $I = \int \frac{\ln(1 + x^2)^x + 2011x}{\ln[(e^{x^2} + e)^{x^2 + 1}]} dx$

5. $J = \int_1^e \frac{xe^x + 1}{x(e^x + \ln x)} dx$

6. $I = \int_0^{\ln 2} \frac{2e^{3x} + e^{2x} - 1}{e^{3x} + e^{2x} - e^x + 1} dx$

7. $I = \int_0^{3 \ln 2} \frac{dx}{(\sqrt[3]{e^x} + 2)^2}$

8. $I = \int_0^{\ln 2} \sqrt[3]{e^x - 1} dx$

9. $I = \int_{3 \ln 2}^{\ln 15} \frac{(e^{2x} - 24e^x) dx}{e^x \sqrt{e^x + 1} + 5e^x - 3\sqrt{e^x + 1} - 15}$

10. $I = \int_{\ln 2}^{\ln 3} \frac{e^{2x} dx}{e^x - 1 + \sqrt{e^x - 2}}$

11. $I = \int_0^{\ln 3} \frac{2e^{3x} - e^{2x}}{e^x \sqrt{4e^x - 3} + 1} dx$

12. $I = \int_{\ln \frac{8}{3}}^{\ln 3} \sqrt{3e^x - 4} dx$

13. $I = \int_0^{\ln 3} \frac{e^x}{\sqrt{(e^x + 1)^3}} dx$

14. $I = \int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}} dx$

15. $I = \int_0^{\ln 2} \sqrt{e^x - 1} dx$

16. $I = \int_1^2 \frac{2^x - 2^{-x}}{4^x + 4^{-x} - 2} dx$

17. $I = \int_0^1 \frac{6^x dx}{9^x + 3 \cdot 6^x + 2 \cdot 4^x}$

Bài giải

1. $I = \int \frac{e^{2x}}{1 + \sqrt{e^x}} dx$

Đặt $t = \sqrt{e^x} \Rightarrow e^x = t^2 \Rightarrow e^x dx = 2t dt$.

$$\Rightarrow I = 2 \int \frac{t^3}{1+t} dt = \frac{2}{3} t^3 - t^2 + 2t - 2 \ln|t+1| + C = \frac{2}{3} e^x \sqrt{e^x} - e^x + 2\sqrt{e^x} - 2 \ln|\sqrt{e^x} + 1| + C$$

2. $I = \int \frac{(x^2 + x)e^x}{x + e^{-x}} dx$

$$I = \int \frac{(x^2 + x)e^x}{x + e^{-x}} dx = \int \frac{xe^x \cdot (x+1)e^x}{xe^x + 1} dx. \text{ Đặt } t = xe^x + 1 \Rightarrow I = xe^x + 1 - \ln|xe^x + 1| + C.$$

3. $I = \int \frac{dx}{\sqrt{e^{2x} + 9}}$

$$\text{Đặt } t = \sqrt{e^{2x} + 9} \Rightarrow I = \int \frac{dt}{t^2 - 9} = \frac{1}{6} \ln \left| \frac{t-3}{t+3} \right| + C = \frac{1}{6} \ln \left| \frac{\sqrt{e^{2x} + 9} - 3}{\sqrt{e^{2x} + 9} + 3} \right| + C$$

$$4. I = \int \frac{\ln(1+x^2)^x + 2011x}{\ln[(e^{x^2} + e)^{x^2+1}]} dx$$

$$\text{Ta có: } I = \int \frac{x[\ln(x^2+1) + 2011]}{(x^2+1)[\ln(x^2+1) + 1]} dx. \text{ Đặt } t = \ln(x^2+1) + 1$$

$$\Rightarrow I = \frac{1}{2} \int \frac{t+2010}{t} dt = \frac{1}{2} t + 1005 \ln|t| + C = \frac{1}{2} \ln(x^2+1) + \frac{1}{2} + 1005 \ln(\ln(x^2+1) + 1) + C$$

$$5. J = \int_1^e \frac{xe^x + 1}{x(e^x + \ln x)} dx \quad J = \int_1^e \frac{d(e^x + \ln x)}{e^x + \ln x} = \ln|e^x + \ln x| \Big|_1^e = \ln \frac{e^e + 1}{e}$$

$$6. I = \int_0^{\ln 2} \frac{2e^{3x} + e^{2x} - 1}{e^{3x} + e^{2x} - e^x + 1} dx$$

$$I = \int_0^{\ln 2} \frac{3e^{3x} + 2e^{2x} - e^x - (e^{3x} + e^{2x} - e^x + 1)}{e^{3x} + e^{2x} - e^x + 1} dx = \int_0^{\ln 2} \left(\frac{3e^{3x} + 2e^{2x} - e^x}{e^{3x} + e^{2x} - e^x + 1} - 1 \right) dx$$

$$= \ln(e^{3x} + e^{2x} - e^x + 1) \Big|_0^{\ln 2} - x \Big|_0^{\ln 2} = \ln 11 - \ln 4 = \ln \frac{11}{4}$$

$$7. I = \int_0^{3 \ln 2} \frac{dx}{(\sqrt[3]{e^x} + 2)^2}$$

$$I = \int_0^{3 \ln 2} \frac{e^{\frac{x}{3}} dx}{e^{\frac{x}{3}} (\sqrt[3]{e^x} + 2)^2}. \text{ Đặt } t = e^{\frac{x}{3}} \Rightarrow dt = \frac{1}{3} e^{\frac{x}{3}} dx \Rightarrow I = \frac{3}{4} \left(\ln \frac{3}{2} - \frac{1}{6} \right)$$

$$8. I = \int_0^{\ln 2} \sqrt[3]{e^x - 1} dx$$

$$\text{Đặt } \sqrt[3]{e^x - 1} = t \Rightarrow dx = \frac{3t^2 dt}{t^3 + 1} \Rightarrow I = 3 \int_0^1 \left(1 - \frac{1}{t^3 + 1} \right) dt = 3 - 3 \int_0^1 \frac{dt}{t^3 + 1}$$

$$\text{Tính } I_1 = 3 \int_0^1 \frac{dt}{t^3 + 1} = \int_0^1 \left(\frac{1}{t+1} + \frac{2-t}{t^2 - t + 1} \right) dt = \frac{\pi}{\sqrt{3}} + \ln 2$$

$$\text{Vậy: } I = 3 - \ln 2 - \frac{\pi}{\sqrt{3}}$$

$$9. I = \int_{3 \ln 2}^{\ln 15} \frac{(e^{2x} - 24e^x) dx}{e^x \sqrt{e^x + 1} + 5e^x - 3\sqrt{e^x + 1} - 15}$$

$$\text{Đặt } t = \sqrt{e^x + 1} \Rightarrow t^2 - 1 = e^x \Rightarrow e^x dx = 2t dt.$$

$$I = \int_3^4 \frac{(2t^2 - 10t)dt}{t^2 - 4} = \int_3^4 \left(2 - \frac{3}{t-2} - \frac{7}{t+2} \right) dt = (2t - 3 \ln|t-2| - 7 \ln|t+2|) \Big|_3^4 = 2 - 3 \ln 2 - 7 \ln 6 + 7 \ln 5$$

$$10. I = \int_{\ln 2}^{\ln 3} \frac{e^{2x} dx}{\ln 2 e^x - 1 + \sqrt{e^x - 2}}$$

$$\text{Đặt } t = \sqrt{e^x - 2} \Rightarrow e^{2x} dx = 2t dt$$

$$\Rightarrow I = 2 \int_0^1 \frac{(t^2 + 2)t dt}{t^2 + t + 1} = 2 \int_0^1 \left(t - 1 + \frac{2t + 1}{t^2 + t + 1} \right) dt = 2 \int_0^1 (t - 1) dt + 2 \int_0^1 \frac{d(t^2 + t + 1)}{t^2 + t + 1}$$

$$= (t^2 - 2t) \Big|_0^1 + 2 \ln(t^2 + t + 1) \Big|_0^1 = 2 \ln 3 - 1.$$

$$11. I = \int_0^{\ln 3} \frac{2e^{3x} - e^{2x}}{e^x \sqrt{4e^x - 3} + 1} dx$$

$$\text{Đặt } t = \sqrt{4e^{3x} - 3e^{2x}} \Rightarrow t^2 = 4e^{3x} - 3e^{2x} \Rightarrow 2t dt = (12e^{3x} - 6e^{2x}) dx \Rightarrow (2e^{3x} - e^{2x}) dx = \frac{t dt}{3}$$

$$\Rightarrow I = \frac{1}{3} \int_1^9 \frac{t dt}{t + 1} = \frac{1}{3} \int_1^9 \left(1 - \frac{1}{t + 1} \right) dt = \frac{1}{3} (t - \ln|t + 1|) \Big|_1^9 = \frac{8 - \ln 5}{3}.$$

$$12. I = \int_{\ln \frac{8}{3}}^{\ln \frac{16}{3}} \sqrt{3e^x - 4} dx$$

$$\text{Đặt: } t = \sqrt{3e^x - 4} \Rightarrow e^x = \frac{t^2 + 4}{3} \Rightarrow dx = \frac{2t dt}{t^2 + 4}$$

$$\Rightarrow I = \int_2^{2\sqrt{3}} \frac{2t^2}{t^2 + 4} dt = 2 \int_2^{2\sqrt{3}} dt - 8 \int_2^{2\sqrt{3}} \frac{dt}{t^2 + 4} = 4(\sqrt{3} - 1) - 8I_1, \text{ với } I_1 = \int_2^{2\sqrt{3}} \frac{dt}{t^2 + 4}$$

$$\text{Tính } I_1 = \int_2^{2\sqrt{3}} \frac{dt}{t^2 + 4}. \text{ Đặt: } t = 2 \tan u, u \in \left(-\frac{\pi}{2}; \frac{\pi}{2} \right) \Rightarrow dt = 2(1 + \tan^2 u) du$$

$$\Rightarrow I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2} du = \frac{1}{2} \left(\frac{\pi}{3} - \frac{\pi}{4} \right) = \frac{\pi}{24}. \text{ Vậy: } I = 4(\sqrt{3} - 1) - \frac{\pi}{3}$$

$$13. I = \int_0^{\ln 3} \frac{e^x}{\sqrt{(e^x + 1)^3}} dx$$

$$\text{Đặt } t = \sqrt{e^x + 1} \Leftrightarrow t^2 = e^x + 1 \Leftrightarrow 2tdt = e^x dx \Rightarrow dx = \frac{2tdt}{e^x} \Rightarrow I = 2 \int \frac{tdt}{\sqrt{2} t^3} = \sqrt{2} - 1$$

$$14. I = \int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}} dx$$

$$\text{Đặt } t = \sqrt{e^x - 1} \Leftrightarrow t^2 = e^x - 1 \Rightarrow dx = \frac{2tdt}{e^x} \Rightarrow I = 2 \int_1^2 (t^2 + 1) dt = 2 \left(\frac{t^3}{3} + t \right) \Big|_1^2 = \frac{20}{3}$$

$$15. I = \int_0^{\ln 2} \sqrt{e^x - 1} dx$$

$$\text{Đặt } t = \sqrt{e^x - 1} \Rightarrow t^2 = e^x - 1 \Rightarrow 2tdt = e^x dx \Rightarrow dx = \frac{2tdt}{e^x} = \frac{2tdt}{t^2 + 1}$$

$$\Rightarrow I = \int_0^1 \frac{2t^2}{t^2 + 1} dt = 2 \int_0^1 \left(1 - \frac{1}{t^2 + 1} \right) dt = \frac{4 - \pi}{2}$$

$$16. I = \int_1^2 \frac{2^x - 2^{-x}}{4^x + 4^{-x} - 2} dx$$

$$\text{Đặt } t = 2^x + 2^{-x} \Rightarrow 4^x + 4^{-x} - 2 = (2^x + 2^{-x})^2 - 4 \Rightarrow I = \frac{1}{4 \ln 2} \ln \frac{81}{25}$$

$$17. I = \int_0^1 \frac{6^x dx}{9^x + 3 \cdot 6^x + 2 \cdot 4^x}$$

$$\text{Ta có: } I = \int_0^1 \frac{\left(\frac{3}{2}\right)^x dx}{\left(\frac{3}{2}\right)^{2x} + 3\left(\frac{3}{2}\right)^x + 2} \cdot \text{Đặt } t = \left(\frac{3}{2}\right)^x \cdot I = \frac{1}{\ln 3 - \ln 2} \int_1^{\frac{3}{2}} \frac{dt}{t^2 + 3t + 2} = \frac{\ln 15 - \ln 14}{\ln 3 - \ln 2}$$

HT 2. Tính các tích phân sau:

$$1. I = \int_1^e \left(\frac{\ln x}{x\sqrt{1 + \ln x}} + 3x^2 \ln x \right) dx$$

$$2. I = \int_1^e \frac{\ln x \sqrt[3]{2 + \ln^2 x}}{x} dx$$

$$3. I = \int_e^{e^2} \frac{dx}{x \ln x \cdot \ln ex}$$

$$4. I = \int_{\ln 4}^{\ln 6} \frac{e^{2x}}{e^x + 6e^{-x} - 5} dx$$

$$9. I = \int_2^5 \frac{\ln(\sqrt{x-1} + 1)}{x-1 + \sqrt{x-1}} dx$$

$$10. I = \int_1^{e^3} \frac{\ln^3 x}{x\sqrt{1 + \ln x}} dx$$

$$11. I = \int_1^{\sqrt{e}} \frac{3 - 2 \ln x}{x\sqrt{1 + 2 \ln x}} dx$$

$$12. I = \int_1^e \frac{\ln x \sqrt[3]{2 + \ln^2 x}}{x} dx$$

$$5. I = \int_1^e \frac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}} dx$$

$$6. I = \int_1^e \frac{x + (x-2)\ln x}{x(1+\ln x)} dx$$

$$7. I = \int_{e^2}^{e^3} \frac{2x \ln^2 x - x \ln x^2 + 3}{x(1-\ln x)} dx$$

$$8. I = \int_1^{e^2} \frac{\sqrt{\ln^2 x - \ln x^2 + 1}}{x^2} dx$$

$$13. I = \int_1^e \frac{xe^x + 1}{x(e^x + \ln x)} dx$$

Bài giải

$$1. I = \int_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + 3x^2 \ln x \right) dx$$

$$I = \int_1^e \frac{\ln x}{x\sqrt{1+\ln x}} dx + 3 \int_1^e x^2 \ln x dx = \frac{2(2-\sqrt{2})}{3} + \frac{2e^3+1}{3} = \frac{5-2\sqrt{2}+2e^3}{3}$$

$$2. I = \int_1^e \frac{\ln x \sqrt[3]{2+\ln^2 x}}{x} dx$$

$$\text{Đặt } t = 2 + \ln^2 x \Rightarrow dt = \frac{2 \ln x}{x} dx \Rightarrow I = \frac{1}{2} \int_2^3 \sqrt[3]{t} dt = \frac{3}{8} (\sqrt[3]{3^4} - \sqrt[3]{2^4})$$

$$3. I = \int_e^{e^2} \frac{dx}{x \ln x \cdot \ln ex}$$

$$I = \int_e^{e^2} \frac{dx}{x \ln x (1 + \ln x)} = \int_e^{e^2} \frac{d(\ln x)}{\ln x (1 + \ln x)} = \int_e^{e^2} \left(\frac{1}{\ln x} - \frac{1}{1 + \ln x} \right) d(\ln x) = 2 \ln 2 - \ln 3$$

$$4. I = \int_{\ln 4}^{\ln 6} \frac{e^{2x}}{e^x + 6e^{-x} - 5} dx$$

$$\bullet \text{Đặt } t = e^x. I = 2 + 9 \ln 3 - 4 \ln 2$$

$$5. I = \int_1^e \frac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}} dx$$

$$I = \int_1^e \frac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}} dx = \int_1^e \frac{\left(\frac{\ln x}{\ln 2}\right)^3}{x\sqrt{1+3\ln^2 x}} dx = \frac{1}{\ln^3 2} \int_1^e \frac{\ln^2 x \cdot \ln x dx}{\sqrt{1+3\ln^2 x} \cdot x}$$

$$\text{Đặt } \sqrt{1+3\ln^2 x} = t \Rightarrow \ln^2 x = \frac{1}{3}(t^2 - 1) \Rightarrow \ln x \cdot \frac{dx}{x} = \frac{1}{3} t dt.$$

$$\text{Suy ra } I = \frac{1}{9 \ln^3 2} \left(\frac{1}{3} t^3 - t \right) \Big|_1^2 = \frac{4}{27 \ln^3 2}.$$

$$6. I = \int_1^e \frac{x + (x-2)\ln x}{x(1+\ln x)} dx$$

$$\int_1^e dx - 2 \int_1^e \frac{\ln x}{x(1+\ln x)} dx = e - 1 - 2 \int_1^e \frac{\ln x}{x(1+\ln x)} dx$$

$$\text{Tính } J = \int_1^e \frac{\ln x}{x(1+\ln x)} dx. \text{ Đặt } t = 1 + \ln x \Rightarrow J = \int_1^2 \frac{t-1}{t} dt = 1 - \ln 2.$$

$$\text{Vậy: } I = e - 3 + 2 \ln 2.$$

$$7. I = \int_{e^2}^{e^3} \frac{2x \ln^2 x - x \ln x^2 + 3}{x(1-\ln x)} dx$$

$$I = 3 \int_{e^2}^{e^3} \frac{1}{x(1-\ln x)} dx - 2 \int_{e^2}^{e^3} \ln x dx = -3 \ln 2 - 4e^3 + 2e^2.$$

$$8. I = \int_1^{e^2} \frac{\sqrt{\ln^2 x - \ln x^2 + 1}}{x^2} dx$$

$$\text{Đặt } t = \ln x \Rightarrow dt = \frac{dx}{x} \Rightarrow I = \int_0^2 \frac{\sqrt{t^2 - 2t + 1}}{e^t} dt = \int_0^2 \frac{|t-1|}{e^t} dt = -\int_0^1 \frac{t-1}{e^t} dt + \int_1^2 \frac{t-1}{e^t} dt = I_1 + I_2$$

$$+ I_1 = -\left(\int_0^1 \frac{t dt}{e^t} - \int_0^1 \frac{dt}{e^t}\right) = -\left(-te^{-t}\Big|_0^1 + \int_0^1 \frac{dt}{e^t} - \int_0^1 \frac{dt}{e^t}\right) = \frac{1}{e}$$

$$+ I_2 = \int_1^2 \frac{t dt}{e^t} - \int_1^2 \frac{dt}{e^t} = -te^{-t}\Big|_1^2 + \int_1^2 \frac{dt}{e^t} - \int_1^2 \frac{dt}{e^t} = -te^{-t}\Big|_1^2 = \frac{1}{e} - \frac{2}{e^2}$$

$$\text{Vậy: } I = \frac{2(e-1)}{e^2}$$

$$9. I = \int_2^5 \frac{\ln(\sqrt{x-1}+1)}{x-1+\sqrt{x-1}} dx$$

$$\text{Đặt } t = \ln(\sqrt{x-1}+1) \Rightarrow 2dt = \frac{dx}{x-1+\sqrt{x-1}} \Rightarrow I = 2 \int_{\ln 2}^{\ln 3} dt = \ln^2 3 - \ln^2 2.$$

$$10. I = \int_1^{e^3} \frac{\ln^3 x}{x\sqrt{1+\ln x}} dx$$

$$\text{Đặt } t = \sqrt{1+\ln x} \Rightarrow 1+\ln x = t^2 \Rightarrow \frac{dx}{x} = 2t dt \text{ và } \ln^3 x = (t^2 - 1)^3$$

$$\Rightarrow I = \int_1^2 \frac{(t^2-1)^3}{t} dt = \int_1^2 \frac{t^6 - 3t^4 + 3t^2 - 1}{t} dt = \int_1^2 \left(t^5 - 3t^3 + 3t - \frac{1}{t}\right) dt = \frac{15}{4} - \ln 2$$

$$11. I = \int_1^{\sqrt{e}} \frac{3 - 2 \ln x}{x \sqrt{1 + 2 \ln x}} dx$$

$$\text{Đặt } t = \sqrt{1 + 2 \ln x} \Rightarrow I = \int_1^{\sqrt{e}} (2 - t^2) dt = \frac{4\sqrt{2} - 5}{3}$$

$$12. I = \int_1^e \frac{\ln x \sqrt[3]{2 + \ln^2 x}}{x} dx$$

$$\text{Đặt } t = 2 + \ln^2 x \Rightarrow I = \frac{3}{8} [\sqrt[3]{3^4} - \sqrt[3]{2^4}]$$

$$13. I = \int_1^e \frac{x e^x + 1}{x(e^x + \ln x)} dx$$

$$\text{Đặt } t = e^x + \ln x \Rightarrow I = \ln \frac{e^e + 1}{e}$$

HT 3. Tính các tích phân sau:

$$1. I = \int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \sin 2x dx$$

$$2. I = \int_0^1 x \ln(x^2 + x + 1) dx$$

$$3. I = \int_3^8 \frac{\ln x}{\sqrt{x+1}} dx$$

$$4. I = \int_1^e \frac{x^2 + x \ln x + 1}{x} e^x dx$$

$$5. I = \int_1^e \left(\frac{\ln x}{x \sqrt{1 + \ln x}} + \ln^2 x \right) dx$$

$$6. I = \int_1^2 \frac{\ln(x^2 + 1)}{x^3} dx$$

$$7. I = \int_1^2 \frac{\ln(x+1)}{x^2} dx$$

$$8. I = \int_0^{\frac{1}{2}} x \ln \left(\frac{1+x}{1-x} \right) dx$$

$$9. I = \int_1^2 x^2 \cdot \ln \left(x + \frac{1}{x} \right) dx$$

$$10. I = \int_0^1 x^2 \cdot \ln(1 + x^2) dx$$

$$11. I = \int_1^3 \frac{\ln x}{(x+1)^2} dx$$

$$12. I = \int_1^e \frac{\ln^2 x + e^x (e^x + \ln^2 x)}{1 + e^x} dx$$

$$13. I = \int_{\frac{1}{2}}^2 \left(x + 1 - \frac{1}{x} \right) e^{x+\frac{1}{x}} dx$$

$$14. I = \int_0^4 \ln(\sqrt{x^2 + 9} - x) dx$$

Bài giải

$$1. I = \int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \sin 2x dx$$

$$I = 2 \int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \sin x \cos x dx. \text{ Đặt } \begin{cases} u = \sin x \\ dv = e^{\sin x} \cos x dx \end{cases} \Rightarrow \begin{cases} du = \cos x dx \\ v = e^{\sin x} \end{cases}$$

$$\Rightarrow I = 2 \sin x e^{\sin x} \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^{\sin x} \cdot \cos x dx = 2e - 2e^{\sin x} \Big|_0^{\frac{\pi}{2}} = 2$$

$$2. I = \int_0^1 x \ln(x^2 + x + 1) dx$$

$$\text{Đặt } \begin{cases} u = \ln(x^2 + x + 1) \\ dv = x dx \end{cases} \Rightarrow \begin{cases} du = \frac{2x+1}{x^2+x+1} dx \\ v = \frac{x^2}{2} \end{cases}$$

$$I = \frac{x^2}{2} \ln(x^2 + x + 1) \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{2x^3 + x^2}{x^2 + x + 1} dx = \frac{1}{2} \ln 3 - \frac{1}{2} \int_0^1 (2x-1) dx + \frac{1}{4} \int_0^1 \frac{2x+1}{x^2+x+1} dx - \frac{3}{4} \int_0^1 \frac{dx}{x^2+x+1}$$

$$= \frac{3}{4} \ln 3 - \frac{\sqrt{3}\pi}{12}$$

$$3. I = \int_3^8 \frac{\ln x}{\sqrt{x+1}} dx$$

$$\text{Đặt } \begin{cases} u = \ln x \\ dv = \frac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \frac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = (2\sqrt{x+1} \cdot \ln x) \Big|_3^8 - 2 \int_3^8 \frac{\sqrt{x+1}}{x} dx = 6 \ln 8 - 4 \ln 3 - 2J$$

$$+ \text{Tính } J = \int_3^8 \frac{\sqrt{x+1}}{x} dx. \text{ Đặt } t = \sqrt{x+1} \Rightarrow J = \int_2^3 \frac{t}{t^2-1} \cdot 2t dt = 2 \int_2^3 \frac{t^2}{t^2-1} dt = \int_2^3 \left(2 + \frac{1}{t-1} - \frac{1}{t+1} \right) dt$$

$$= \left(2t + \ln \left| \frac{t-1}{t+1} \right| \right) \Big|_2^3 = 2 + \ln 3 - \ln 2$$

$$\text{Từ đó } I = 20 \ln 2 - 6 \ln 3 - 4.$$

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$$4. I = \int_1^e \frac{x^2 + x \ln x + 1}{x} e^x dx$$

$$I = \int_1^e x e^x dx + \int_1^e \ln x e^x dx + \int_1^e \frac{e^x}{x} dx. \quad + \text{Tính } I_1 = \int_1^e x e^x dx = x e^x \Big|_1^e - \int_1^e e^x dx = e^e (e-1)$$

$$+ \text{Tính } I_2 = \int_1^e x \ln x dx = x \ln x \Big|_1^e - \int_1^e \frac{e^x}{x} dx = e^e - \int_1^e \frac{e^x}{x} dx.$$

$$\text{Vậy: } I = I_1 + I_2 + \int_1^e \frac{e^x}{x} dx = e^{e+1}.$$

$$5. I = \int_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + \ln^2 x \right) dx$$

$$\text{Tính } I_1 = \int_1^e \frac{\ln x}{x\sqrt{1+\ln x}} dx. \text{ Đặt } t = \sqrt{1+\ln x} \Rightarrow I_1 = \frac{4}{3} - \frac{2\sqrt{2}}{3}.$$

$$+ \text{Tính } I_2 = \int_1^e \ln^2 x dx. \text{ Lấy tích phân từng phần 2 lần được } I_2 = e - 2.$$

$$\text{Vậy } I = e - \frac{2}{3} - \frac{2\sqrt{2}}{3}.$$

$$6. I = \int_1^2 \frac{\ln(x^2+1)}{x^3} dx$$

$$\text{Đặt } \begin{cases} u = \ln(x^2+1) \\ dv = \frac{dx}{x^3} \end{cases} \Rightarrow \begin{cases} du = \frac{2x}{x^2+1} \\ v = -\frac{1}{2x^2} \end{cases}. \text{ Do đó } I = -\frac{\ln(x^2+1)}{2x^2} \Big|_1^2 + \int_1^2 \frac{dx}{x(x^2+1)}$$

$$= \frac{\ln 2}{2} - \frac{\ln 5}{8} + \int_1^2 \left(\frac{1}{x} - \frac{x}{x^2+1} \right) dx = \frac{\ln 2}{2} - \frac{\ln 5}{8} + \int_1^2 \frac{dx}{x} - \frac{1}{2} \int_1^2 \frac{d(x^2+1)}{x^2+1}$$

$$= \frac{\ln 2}{2} - \frac{\ln 5}{8} + \left(\ln|x| - \frac{1}{2} \ln|x^2+1| \right) \Big|_1^2 = 2\ln 2 - \frac{5}{8} \ln 5$$

$$7. I = \int_1^2 \frac{\ln(x+1)}{x^2} dx$$

$$\text{Đặt } \begin{cases} u = \ln(x+1) \\ dv = \frac{dx}{x^2} \end{cases} \Leftrightarrow \begin{cases} du = \frac{dx}{x+1} \\ v = -\frac{1}{x} \end{cases} \Rightarrow I = -\frac{1}{x} \ln(x+1) \Big|_1^2 + \int_1^2 \frac{dx}{(x+1)x} = 3\ln 2 - \frac{3}{2} \ln 3$$

$$8. I = \int_0^{\frac{1}{2}} x \ln \left(\frac{1+x}{1-x} \right) dx$$

$$\text{Đặt } \begin{cases} u = \ln \frac{1+x}{1-x} \\ dv = x dx \end{cases} \Rightarrow \begin{cases} du = \frac{2}{(1-x)^2} dx \\ v = \frac{x^2}{2} \end{cases} \Rightarrow I = \frac{1}{2} \left[x^2 \ln \left(\frac{1+x}{1-x} \right) \Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} x^2 \left(\frac{2}{1-x^2} \right) dx \right]$$

$$= \frac{\ln 3}{8} + \int_0^{\frac{1}{2}} \frac{x^2}{x^2-1} dx = \frac{\ln 3}{8} + \int_0^{\frac{1}{2}} \left[1 + \frac{1}{(x-1)(x+1)} \right] dx = \frac{\ln 3}{8} + \frac{1}{2} + \frac{1}{2} \ln \frac{2}{3}$$

$$9. I = \int_1^2 x^2 \cdot \ln \left(x + \frac{1}{x} \right) dx$$

$$\text{Đặt } \begin{cases} u = \ln \left(x + \frac{1}{x} \right) \\ dv = x^2 dx \end{cases} \Rightarrow I = 3 \ln 3 - \frac{10}{3} \ln 2 + \frac{1}{6}$$

$$10. I = \int_0^1 x^2 \cdot \ln(1+x^2) dx$$

$$\text{Đặt } \begin{cases} u = \ln(1+x^2) \\ dv = x^2 dx \end{cases} \Rightarrow I = \frac{1}{3} \cdot \ln 2 + \frac{4}{9} + \frac{\pi}{6}$$

$$11. I = \int_1^3 \frac{\ln x}{(x+1)^2} dx$$

$$\text{Đặt } \begin{cases} u = \ln x \\ dv = \frac{dx}{(x+1)^2} \end{cases} \Rightarrow I = -\frac{1}{4} \ln 3 + \ln \frac{3}{2}$$

$$12. I = \int_1^e \frac{\ln^2 x + e^x (e^x + \ln^2 x)}{1+e^x} dx$$

$$\text{Ta có: } I = \int_1^e \ln^2 x dx + \int_1^e \frac{e^{2x}}{e^x+1} dx = H + K$$

$$+ H = \int_1^e \ln^2 x dx. \text{ Đặt: } \begin{cases} u = \ln^2 x \\ dv = dx \end{cases} \Rightarrow H = e - \int_1^e 2 \ln x dx = e - 2$$

$$+ K = \int_1^e \frac{e^{2x}}{e^x+1} dx. \text{ Đặt } t = e^x + 1 \Rightarrow I_2 = \int_{e+1}^{e^e+1} \frac{t-1}{t} dt = e^e - e + \ln \frac{e+1}{e^e+1}$$

$$\text{Vậy: } I = e^e - 2 + \ln \frac{e+1}{e^e+1}$$

$$13. I = \int_{\frac{1}{2}}^2 \left(x+1 - \frac{1}{x} \right) e^{x+\frac{1}{x}} dx$$

Ta có:
$$I = \int_{\frac{1}{2}}^2 e^{x+\frac{1}{x}} dx + \int_{\frac{1}{2}}^3 \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx = H + K$$

+ Tính H theo phương pháp từng phần $I_1 = H = xe^{x+\frac{1}{x}} \Big|_{\frac{1}{2}}^2 - \int_{\frac{1}{2}}^2 \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx = \frac{3}{2}e^{\frac{5}{2}} - K$

$$\Rightarrow I = \frac{3}{2}e^{\frac{5}{2}}.$$

14.
$$I = \int_0^4 \ln(\sqrt{x^2+9} - x) dx$$

Đặt $\begin{cases} u = \ln(\sqrt{x^2+9} - x) \\ dv = dx \end{cases} \Rightarrow I = x \ln(\sqrt{x^2+9} - x) \Big|_0^4 + \int_0^4 \frac{x}{\sqrt{x^2+9}} dx = 2$

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PHẦN VI TỔNG HỢP

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HT 1. Tính các tích phân sau:

$$1. I = \int_0^1 \left(x^2 e^{x^3} + \frac{\sqrt[4]{x}}{1 + \sqrt{x}} \right) dx$$

$$2. I = \int_1^2 x \left(e^x - \frac{\sqrt{4-x^2}}{x^3} \right) dx$$

$$3. I = \int_0^1 \frac{x}{\sqrt{4-x^2}} \left(e^{2x} \cdot \sqrt{4-x^2} - x^2 \right) dx.$$

$$4. I = \int_0^1 \frac{x^2 + 1}{(x+1)^2} e^x dx$$

$$5. I = \int_0^{\sqrt{3}} \frac{x^3 \cdot e^{\sqrt{x^2+1}} dx}{\sqrt{1+x^2}}$$

$$6. I = \int \frac{x \ln(x^2 + 1) + x^3}{x^2 + 1} dx$$

$$7. I = \int_0^4 \frac{\ln(x + \sqrt{x^2 + 9}) - 3x^3}{\sqrt{x^2 + 9}} dx$$

$$8. I = \int_1^e \frac{(x^3 + 1) \ln x + 2x^2 + 1}{2 + x \ln x} dx$$

$$9. I = \int_1^{e^3} \frac{\ln^3 x}{x \sqrt{1 + \ln x}} dx$$

$$10. I = \int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^2 x} dx$$

$$1. I = \int_0^1 \left(x^2 e^{x^3} + \frac{\sqrt[4]{x}}{1 + \sqrt{x}} \right) dx$$

$$I = \int_0^1 x^2 e^{x^3} dx + \int_0^1 \frac{\sqrt[4]{x}}{1 + \sqrt{x}} dx.$$

$$+ \text{Tính } I_1 = \int_0^1 x^2 e^{x^3} dx. \text{ Đặt } t = x^3 \Rightarrow I_1 = \frac{1}{3} \int_0^1 e^t dt = \frac{1}{3} e^t \Big|_0^1 = \frac{1}{3} e - \frac{1}{3}.$$

$$+ \text{Tính } I_2 = \int_0^1 \frac{\sqrt[4]{x}}{1 + \sqrt{x}} dx. \text{ Đặt } t = \sqrt[4]{x} \Rightarrow I_2 = 4 \int_0^1 \frac{t^4}{1 + t^2} dt = 4 \left(-\frac{2}{3} + \frac{\pi}{4} \right)$$

$$\text{Vậy: } I = \frac{1}{3} e + \pi - 3$$

$$2. I = \int_1^2 x \left(e^x - \frac{\sqrt{4-x^2}}{x^3} \right) dx$$

$$I = \int_1^2 x e^x dx + \int_1^2 \frac{\sqrt{4-x^2}}{x^2} dx.$$

$$+ \text{Tính } I_1 = \int_1^2 x e^x dx = e^2$$

$$+ \text{Tính } I_2 = \int_1^2 \frac{\sqrt{4-x^2}}{x^2} dx. \text{ Đặt } x = 2 \sin t, t \in \left[0; \frac{\pi}{2} \right].$$

$$\Rightarrow I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^2 t}{\sin^2 t} dt = (-\cot t - t) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \sqrt{3} - \frac{\pi}{3}$$

$$\text{Vậy: } I = e^2 + \sqrt{3} - \frac{\pi}{3}.$$

$$3. I = \int_0^1 \frac{x}{\sqrt{4-x^2}} (e^{2x} \sqrt{4-x^2} - x^2) dx.$$

$$I = \int_0^1 x e^{2x} dx - \int_0^1 \frac{x^3}{\sqrt{4-x^2}} dx = I_1 + I_2$$

$$+ \text{Tính } I_1 = \int_0^1 x e^{2x} dx = \frac{e^2 + 1}{4}$$

$$+ \text{Tính } I_2 = \int_0^1 \frac{x^3}{\sqrt{4-x^2}} dx. \text{ Đặt } t = \sqrt{4-x^2} \Rightarrow I_2 = -3\sqrt{3} + \frac{16}{3}$$

$$\Rightarrow I = \frac{e^2}{4} + 3\sqrt{3} - \frac{61}{12}$$

$$4. I = \int_0^1 \frac{x^2 + 1}{(x+1)^2} e^x dx$$

$$\text{Đặt } t = x+1 \Rightarrow dx = dt \quad I = \int_1^2 \frac{t^2 - 2t + 2}{t^2} e^{t-1} dt = \int_1^2 \left(1 + \frac{2}{t^2} - \frac{2}{t}\right) e^{t-1} dt = e - 1 + \frac{2}{e} \left(-\frac{e^2}{2} + e\right) = 1$$

$$5. I = \int_0^{\sqrt{3}} \frac{x^3 \cdot e^{\sqrt{x^2+1}} dx}{\sqrt{1+x^2}}$$

$$\text{Đặt } t = \sqrt{1+x^2} \Rightarrow dx = t dt \Rightarrow I = \int_1^2 (t^2 - 1) e^t dt = \int_1^2 t^2 e^t dt - e^t \Big|_1^2 = J - (e^2 - e)$$

$$+ J = \int_1^2 t^2 e^t dt = t^2 e^t \Big|_1^2 - \int_1^2 2te^t dt = 4e^2 - e - 2 \left(te^t \Big|_1^2 - \int_1^2 e^t dt \right) = 4e^2 - e - 2(te^2 - e^2) \Big|_1^2$$

$$\text{Vậy: } I = e^2$$

$$6. I = \int \frac{x \ln(x^2 + 1) + x^3}{x^2 + 1} dx$$

$$\text{Ta có: } f(x) = \frac{x \ln(x^2 + 1)}{x^2 + 1} + \frac{x(x^2 + 1) - x}{x^2 + 1} = \frac{x \ln(x^2 + 1)}{x^2 + 1} + x - \frac{x}{x^2 + 1}$$

$$\Rightarrow F(x) = \int f(x) dx = \frac{1}{2} \int \ln(x^2 + 1) d(x^2 + 1) + \int x dx - \frac{1}{2} \int d \ln(x^2 + 1)$$

$$= \frac{1}{4} \ln^2(x^2 + 1) + \frac{1}{2} x^2 - \frac{1}{2} \ln(x^2 + 1) + C.$$

$$7. I = \int_0^4 \frac{\ln(x + \sqrt{x^2 + 9}) - 3x^3}{\sqrt{x^2 + 9}} dx$$

$$I = \int_0^4 \frac{\ln(x + \sqrt{x^2 + 9}) - 3x^3}{\sqrt{x^2 + 9}} dx = \int_0^4 \frac{\ln(x + \sqrt{x^2 + 9})}{\sqrt{x^2 + 9}} dx - 3 \int_0^4 \frac{x^3}{\sqrt{x^2 + 9}} dx = I_1 - 3I_2$$

$$+ \text{Tính } I_1 = \int_0^4 \frac{\ln(x + \sqrt{x^2 + 9})}{\sqrt{x^2 + 9}} dx. \text{ Đặt } \ln(x + \sqrt{x^2 + 9}) = u \Rightarrow du = \frac{1}{\sqrt{x^2 + 9}} dx$$

$$\Rightarrow I_1 = \int_{\ln 3}^{\ln 9} u du = \frac{u^2}{2} \Big|_{\ln 3}^{\ln 9} = \frac{\ln^2 9 - \ln^2 3}{2}$$

$$+ \text{Tính } I_2 = \int_0^4 \frac{x^3}{\sqrt{x^2 + 9}} dx. \text{ Đặt } \sqrt{x^2 + 9} = v \Rightarrow dv = \frac{x}{\sqrt{x^2 + 9}} dx, x^2 = v^2 - 9$$

$$\Rightarrow I_2 = \int_3^5 (u^2 - 9) du = \left(\frac{u^3}{3} - 9u \right) \Big|_3^5 = \frac{44}{3}$$

$$\text{Vậy } I = \int_0^4 \frac{\ln(x + \sqrt{x^2 + 9}) - 3x^3}{\sqrt{x^2 + 9}} dx = I_1 - 3I_2 = \frac{\ln^2 9 - \ln^2 3}{2} - 44.$$

$$8. I = \int_1^e \frac{(x^3 + 1) \ln x + 2x^2 + 1}{2 + x \ln x} dx$$

$$I = \int_1^e x^2 dx + \int_1^e \frac{1 + \ln x}{2 + x \ln x} dx. \quad + \int_1^e x^2 dx = \frac{x^3}{3} \Big|_1^e = \frac{e^3 - 1}{3}$$

$$+ \int_1^e \frac{1 + \ln x}{2 + x \ln x} dx = \int_1^e \frac{d(2 + x \ln x)}{2 + x \ln x} = \ln|2 + x \ln x| \Big|_1^e = \ln \frac{e + 2}{2}. \quad \text{Vậy: } I = \frac{e^3 - 1}{3} + \ln \frac{e + 2}{2}.$$

$$9. I = \int_1^{e^3} \frac{\ln^3 x}{x \sqrt{1 + \ln x}} dx$$

$$\text{Đặt } t = \sqrt{1 + \ln x} \Rightarrow 1 + \ln x = t^2 \Rightarrow \frac{dx}{x} = 2t dt \text{ và } \ln^3 x = (t^2 - 1)^3$$

$$\Rightarrow I = \int_1^2 \frac{(t^2 - 1)^3}{t} dt = \int_1^2 \frac{t^6 - 3t^4 + 3t^2 - 1}{t} dt = \int_1^2 (t^5 - 3t^3 + 3t - \frac{1}{t}) dt = \frac{15}{4} - \ln 2$$

$$10. I = \int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^2 x} dx$$

$$\text{Đặt } \begin{cases} u = x \\ dv = \frac{\sin x}{\cos^2 x} dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{\cos x} \end{cases} \Rightarrow I = \frac{x}{\cos x} \Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \frac{\pi\sqrt{2}}{4} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x}$$

$$+ I_1 = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \int_0^{\frac{\pi}{4}} \frac{\cos x dx}{1 - \sin^2 x}. \text{Đặt } t = \sin x \Rightarrow I_1 = \int_0^{\frac{\sqrt{2}}{2}} \frac{dt}{1-t^2} = \frac{1}{2} \ln \frac{2+\sqrt{2}}{2-\sqrt{2}}$$

$$\text{Vậy: } = \frac{\pi\sqrt{2}}{4} - \frac{1}{2} \ln \frac{2+\sqrt{2}}{2-\sqrt{2}}$$

HT 2. Tính các tích phân sau:

$$1. I = \int_1^4 \frac{\ln(5-x) + x^3 \cdot \sqrt{5-x}}{x^2} dx$$

$$2. I = \int_0^2 [\sqrt{x(2-x)} + \ln(4+x^2)] dx$$

$$3. I = \int_3^8 \frac{\ln x}{\sqrt{x+1}} dx$$

$$4. I = \int_1^2 \frac{1+x^2}{x^3} \ln x dx$$

$$5. I = \int_1^e \frac{x^2 + x \ln x + 1}{x} e^x dx$$

$$6. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^3 x} dx$$

$$7. I = \int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^3 x} dx$$

$$8. I = \int_0^{\frac{\pi}{2}} \frac{(x + \sin^2 x) dx}{1 + \sin 2x}$$

$$9. I = \int_0^{\pi} \frac{x(\cos^3 x + \cos x + \sin x)}{1 + \cos^2 x} dx$$

$$10. I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x + \sin x) \sin x}{(1 + \sin x) \sin^2 x} dx$$

Bài giải

$$1. I = \int_1^4 \frac{\ln(5-x) + x^3 \cdot \sqrt{5-x}}{x^2} dx$$

$$\text{Ta có: } I = \int_1^4 \frac{\ln(5-x)}{x^2} dx + \int_1^4 x\sqrt{5-x} dx = K + H.$$

$$+ K = \int_1^4 \frac{\ln(5-x)}{x^2} dx. \text{Đặt } \begin{cases} u = \ln(5-x) \\ dv = \frac{dx}{x^2} \end{cases} \Rightarrow K = \frac{3}{5} \ln 4$$

$$+ H = \int_1^4 x\sqrt{5-x} dx. \text{Đặt } t = \sqrt{5-x} \Rightarrow H = \frac{164}{15}$$

$$\text{Vậy: } I = \frac{3}{5} \ln 4 + \frac{164}{15}$$

$$2. I = \int_0^2 [\sqrt{x(2-x)} + \ln(4+x^2)] dx$$

$$\text{Ta có: } I = \int_0^2 \sqrt{x(2-x)} dx + \int_0^2 \ln(4+x^2) dx = I_1 + I_2$$

$$+ I_1 = \int_0^2 \sqrt{x(2-x)} dx = \int_0^2 \sqrt{1-(x-1)^2} dx = \frac{\pi}{2} \quad (\text{sử dụng đổi biến: } x = 1 + \sin t)$$

$$+ I_2 = \int_0^2 \ln(4+x^2) dx = x \ln(4+x^2) \Big|_0^2 - 2 \int_0^2 \frac{x^2}{4+x^2} dx \quad (\text{sử dụng tích phân từng phần})$$

$$= 6 \ln 2 + \pi - 4 \quad (\text{đổi biến } x = 2 \tan t)$$

$$\text{Vậy: } I = I_1 + I_2 = \frac{3\pi}{2} - 4 + 6 \ln 2$$

$$3. I = \int_3^8 \frac{\ln x}{\sqrt{x+1}} dx$$

$$\text{Đặt } \begin{cases} u = \ln x \\ dv = \frac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \frac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = 2\sqrt{x+1} \ln x \Big|_3^8 - 2 \int_3^8 \frac{\sqrt{x+1}}{x} dx$$

$$+ \text{Tính } J = \int_3^8 \frac{\sqrt{x+1}}{x} dx. \text{ Đặt } t = \sqrt{x+1} \Rightarrow J = \int_2^3 \frac{2t^2 dt}{t^2-1} = 2 \int_2^3 \left(1 + \frac{1}{t^2-1} \right) dt = 2 + \ln 3 - \ln 2$$

$$\Rightarrow I = 6 \ln 8 - 4 \ln 3 - 2(2 + \ln 3 - \ln 2) = 20 \ln 2 - 6 \ln 3 - 4$$

$$4. I = \int_1^2 \frac{1+x^2}{x^3} \ln x dx$$

$$\text{Ta có: } I = \int_1^2 \left(\frac{1}{x^3} + \frac{1}{x} \right) \ln x dx. \text{ Đặt } \begin{cases} u = \ln x \\ dv = \left(\frac{1}{x^3} + \frac{1}{x} \right) dx \end{cases}$$

$$\Rightarrow I = \left(\frac{-1}{4x^4} + \ln x \right) \ln x \Big|_1^2 - \int_1^2 \left(\frac{-1}{4x^5} + \frac{1}{x} \ln x \right) dx = -\frac{1}{64} \ln 2 + \frac{63}{4} + \frac{1}{2} \ln^2 2$$

$$5. I = \int_1^e \frac{x^2 + x \ln x + 1}{x} e^x dx$$

$$\text{Ta có: } I = \int_1^e x e^x dx + \int_1^e e^x \ln x dx + \int_1^e \frac{e^x}{x} dx = H + K + J$$

$$+ H = \int_1^e x e^x dx = x e^x \Big|_1^e - \int_1^e e^x dx = e^e (e-1)$$

$$+ K = \int_1^e e^x \ln x dx = e^x \ln x \Big|_1^e - \int_1^e \frac{e^x}{x} dx = e^e - \int_1^e \frac{e^x}{x} dx = e^e - J$$

$$\text{Vậy: } I = H + K + J = e^{e+1} - e^e + e^e - J + J = e^{e+1}.$$

$$6. I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^3 x} dx$$

$$\text{Ta có } \left(\frac{1}{\sin^2 x} \right)' = -\frac{2 \cos x}{\sin^3 x}. \text{ Đặt } \begin{cases} u = x \\ dv = \frac{\cos x}{\sin^3 x} dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\frac{1}{2 \sin^2 x} \end{cases}$$

$$\Rightarrow I = -\frac{1}{2} x \cdot \frac{1}{\sin^2 x} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\sin^2 x} = -\frac{1}{2} \left(\frac{\pi}{2} - \frac{\pi}{2} \right) - \frac{1}{2} \cot x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2}.$$

$$7. I = \int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^3 x} dx$$

$$\text{Đặt: } \begin{cases} u = x \\ dv = \frac{\sin x}{\cos^3 x} dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{2 \cdot \cos^2 x} \end{cases} \Rightarrow I = \frac{x}{2 \cos^2 x} \Big|_0^{\frac{\pi}{4}} - \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \frac{\pi}{4} - \frac{1}{2} \tan x \Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \frac{1}{2}$$

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$$8. I = \int_0^{\frac{\pi}{2}} \frac{(x + \sin^2 x)}{1 + \sin 2x} dx$$

$$\text{Ta có: } I = \int_0^{\frac{\pi}{2}} \frac{x}{1 + \sin 2x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x} dx = H + K$$

$$+ H = \int_0^{\frac{\pi}{2}} \frac{x}{1 + \sin 2x} dx = \int_0^{\frac{\pi}{2}} \frac{x}{2 \cos^2 \left(x - \frac{\pi}{4} \right)} dx. \text{ Đặt: } \begin{cases} u = x \\ dv = \frac{dx}{2 \cos^2 \left(x - \frac{\pi}{4} \right)} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{2} \tan \left(x - \frac{\pi}{4} \right) \end{cases}$$

$$\Rightarrow H = \frac{x}{2} \tan \left(x - \frac{\pi}{4} \right) \Big|_0^{\frac{\pi}{2}} + \left(\frac{1}{2} \ln \left| \cos \left(x - \frac{\pi}{4} \right) \right| \right) \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

$$+ K = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin 2x} dx. \text{ Đặt } t = \frac{\pi}{2} - x \Rightarrow K = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + \sin 2x} dx$$

$$\Rightarrow 2K = \int_0^{\frac{\pi}{2}} \frac{dx}{2 \cos^2 \left(x - \frac{\pi}{4} \right)} = \frac{1}{2} \tan \left(x - \frac{\pi}{4} \right) \Big|_0^{\frac{\pi}{2}} = 1 \Rightarrow K = \frac{1}{2}$$

Vậy, $I = H + K = \frac{\pi}{4} + \frac{1}{2}$.

9. $I = \int_0^{\pi} \frac{x(\cos^3 x + \cos x + \sin x)}{1 + \cos^2 x} dx$

Ta có: $I = \int_0^{\pi} x \left(\frac{\cos x(1 + \cos^2 x) + \sin x}{1 + \cos^2 x} \right) dx = \int_0^{\pi} x \cdot \cos x dx + \int_0^{\pi} \frac{x \cdot \sin x}{1 + \cos^2 x} dx = J + K$

+ Tính $J = \int_0^{\pi} x \cdot \cos x dx$. Đặt $\begin{cases} u = x \\ dv = \cos x dx \end{cases} \Rightarrow J = (x \cdot \sin x) \Big|_0^{\pi} - \int_0^{\pi} \sin x dx = 0 + \cos x \Big|_0^{\pi} = -2$

+ Tính $K = \int_0^{\pi} \frac{x \cdot \sin x}{1 + \cos^2 x} dx$. Đặt $x = \pi - t \Rightarrow dx = -dt$

$$\Rightarrow K = \int_0^{\pi} \frac{(\pi - t) \cdot \sin(\pi - t)}{1 + \cos^2(\pi - t)} dt = \int_0^{\pi} \frac{(\pi - t) \cdot \sin t}{1 + \cos^2 t} dt = \int_0^{\pi} \frac{(\pi - x) \cdot \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow 2K = \int_0^{\pi} \frac{(x + \pi - x) \cdot \sin x}{1 + \cos^2 x} dx = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} \Rightarrow K = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x}$$

Đặt $t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow K = \frac{\pi}{2} \int_{-1}^1 \frac{dt}{1 + t^2}$, đặt $t = \tan u \Rightarrow dt = (1 + \tan^2 u) du$

$$\Rightarrow K = \frac{\pi}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1 + \tan^2 u) du}{1 + \tan^2 u} = \frac{\pi}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} du = \frac{\pi}{2} \cdot u \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi^2}{4}$$

Vậy $I = \frac{\pi^2}{4} - 2$

10. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x + \sin x) \sin x}{(1 + \sin x) \sin^2 x} dx$

Ta có: $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x(1 + \sin x) + \sin^2 x}{(1 + \sin x) \sin^2 x} dx = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1 + \sin x} = H + K$

$$+ H = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx. \text{ Đặt } \begin{cases} u = x \\ dv = \frac{dx}{\sin^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x \end{cases} \Rightarrow H = \frac{\pi}{\sqrt{3}}$$

$$+ K = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1 + \sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1 + \cos\left(\frac{\pi}{2} - x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2 \cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)} = \sqrt{3} - 2$$

$$\text{Vậy } I = \frac{\pi}{\sqrt{3}} + \sqrt{3} - 2$$

HT 3. Tính các tích phân sau:

$$1. I = \int_0^{\frac{\pi}{3}} \frac{x + \sin^2 x}{1 + \cos 2x} dx$$

$$2. I = \int_0^3 \sqrt{x+1} \sin \sqrt{x+1} dx$$

$$3. I = \int_0^{\frac{\pi}{2}} \frac{1 + \sin x}{1 + \cos x} e^x dx$$

$$4. I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x (1 + \sin 2x)} dx$$

$$5. I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^6 x + \cos^6 x}{6^x + 1} dx$$

$$6. I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^4 x dx}{2^{-x} + 1}$$

$$7. I = \int_1^{e^\pi} \cos(\ln x) dx$$

$$8. I = \int_0^{\frac{\pi}{2}} e^{\sin^2 x} \cdot \sin x \cdot \cos^3 x dx$$

$$9. I = \int_0^{\frac{\pi}{4}} \ln(1 + \tan x) dx$$

$$10. I = \int_0^{\frac{\pi}{2}} \sin x \ln(1 + \sin x) dx$$

Bài giải

$$1. I = \int_0^{\frac{\pi}{3}} \frac{x + \sin^2 x}{1 + \cos 2x} dx$$

$$\text{Ta có: } I = \int_0^{\frac{\pi}{3}} \frac{x + \sin^2 x}{1 + \cos 2x} dx = \int_0^{\frac{\pi}{3}} \frac{x}{2 \cos^2 x} dx + \int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{2 \cos^2 x} dx = H + K$$

$$+ H = \int_0^{\frac{\pi}{3}} \frac{x}{2 \cos^2 x} dx = \frac{1}{2} \int_0^{\frac{\pi}{3}} \frac{x}{\cos^2 x} dx. \text{ Đặt } \begin{cases} u = x \\ dv = \frac{dx}{\cos^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \tan x \end{cases}$$

$$\Rightarrow H = \frac{1}{2} \left[x \tan x \Big|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan x dx \right] = \frac{\pi}{2\sqrt{3}} + \frac{1}{2} \ln |\cos x| \Big|_0^{\frac{\pi}{3}} = \frac{\pi}{2\sqrt{3}} - \frac{1}{2} \ln 2$$

$$+ K = \int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{2 \cos^2 x} dx = \frac{1}{2} \int_0^{\frac{\pi}{3}} \tan^2 x dx = \frac{1}{2} [\tan x - x] \Big|_0^{\frac{\pi}{3}} = \frac{1}{2} \left(\sqrt{3} - \frac{\pi}{3} \right)$$

$$\text{Vậy: } I = H + K = \frac{\pi}{2\sqrt{3}} - \frac{1}{2} \ln 2 + \frac{1}{2} \left(\sqrt{3} - \frac{\pi}{3} \right) = \frac{\pi(\sqrt{3}-1)}{6} + \frac{1}{2}(\sqrt{3} - \ln 2)$$

$$2. I = \int_0^3 \sqrt{x+1} \sin \sqrt{x+1} dx$$

$$\text{Đặt } t = \sqrt{x+1} \Rightarrow I = \int_1^2 t \cdot \sin t \cdot 2t dt = \int_1^2 2t^2 \sin t dt = \int_1^2 2x^2 \sin x dx$$

$$\text{Đặt } \begin{cases} u = 2x^2 \\ dv = \sin x dx \end{cases} \Rightarrow \begin{cases} du = 4x dx \\ v = -\cos x \end{cases} \Rightarrow I = -2x^2 \cos x \Big|_1^2 + \int_1^2 4x \cos x dx$$

$$\text{Đặt } \begin{cases} u = 4x \\ dv = \cos x dx \end{cases} \Rightarrow \begin{cases} du = 4 dx \\ v = \sin x \end{cases} \text{ . Từ đó suy ra kết quả.}$$

$$3. I = \int_0^{\frac{\pi}{2}} \frac{1 + \sin x}{1 + \cos x} e^x dx$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{e^x dx}{\cos^2 \frac{x}{2}} + \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} e^x dx$$

$$+ \text{Tính } I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} e^x dx = \int_0^{\frac{\pi}{2}} \frac{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} e^x dx = \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} e^x dx$$

$$+ \text{Tính } I_2 = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{e^x dx}{\cos^2 \frac{x}{2}} \cdot \text{Đặt } \begin{cases} u = e^x \\ dv = \frac{dx}{2 \cos^2 \frac{x}{2}} \end{cases} \Rightarrow \begin{cases} du = e^x dx \\ v = \tan \frac{x}{2} \end{cases} \Rightarrow I_2 = e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \tan \frac{x}{2} e^x dx$$

$$\text{Do đó: } I = I_1 + I_2 = e^{\frac{\pi}{2}}.$$

$$4. I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x (1 + \sin 2x)} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x (\sin x + \cos x)^2} dx \cdot \text{Đặt } \begin{cases} u = \frac{\cos x}{e^x} \\ dv = \frac{dx}{(\sin x + \cos x)^2} \end{cases} \Rightarrow \begin{cases} du = \frac{-(\sin x + \cos x) dx}{e^x} \\ v = \frac{\sin x}{\sin x + \cos x} \end{cases}$$

$$\Rightarrow I = \frac{\cos x}{e^x} \cdot \frac{\sin x}{\sin x + \cos x} \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x} = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x}$$

$$\text{Đặt } \begin{cases} u_1 = \sin x \\ dv_1 = \frac{dx}{e^x} \end{cases} \Rightarrow \begin{cases} du_1 = \cos x dx \\ v_1 = \frac{-1}{e^x} \end{cases} \Rightarrow I = \sin x \cdot \frac{-1}{e^x} \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{e^x} = \frac{-1}{e^{\frac{\pi}{2}}} + \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{e^x}$$

$$\text{Đặt } \begin{cases} u_2 = \cos x \\ dv_1 = \frac{dx}{e^x} \end{cases} \Rightarrow \begin{cases} du_2 = -\sin x dx \\ v_1 = \frac{-1}{e^x} \end{cases}$$

$$\Rightarrow I = \frac{-1}{e^{\frac{\pi}{2}}} + \cos x \cdot \frac{-1}{e^x} \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x} = \frac{-1}{e^{\frac{\pi}{2}}} + 1 - I \Rightarrow 2I = -e^{-\frac{\pi}{2}} + 1 \Rightarrow I = \frac{-e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}$$

$$5. I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^6 x + \cos^6 x}{6^x + 1} dx$$

$$\text{Đặt } t = -x \Rightarrow dt = -dx \Rightarrow I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^t \frac{\sin^6 t + \cos^6 t}{6^t + 1} dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^x \frac{\sin^6 x + \cos^6 x}{6^x + 1} dx$$

$$\Rightarrow 2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (6^x + 1) \frac{\sin^6 x + \cos^6 x}{6^x + 1} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin^6 x + \cos^6 x) dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{5}{8} + \frac{3}{8} \cos 4x \right) dx = \frac{5\pi}{16}$$

$$\Rightarrow I = \frac{5\pi}{32}$$

$$6. I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^4 x dx}{2^{-x} + 1}$$

$$\text{Ta có: } I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = \int_{-\frac{\pi}{6}}^0 \frac{2^x \sin^4 x dx}{2^x + 1} + \int_0^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = I_1 + I_2$$

$$+ \text{Tính } I_1 = \int_{-\frac{\pi}{6}}^0 \frac{2^x \sin^4 x dx}{2^x + 1}. \text{ Đặt } x = -t \Rightarrow I_1 = - \int_{\frac{\pi}{6}}^0 \frac{2^{-t} \sin^4(-t) dt}{2^{-t} + 1} = \int_{\frac{\pi}{6}}^0 \frac{\sin^4 t}{2^t + 1} dt = \int_0^{\frac{\pi}{6}} \frac{\sin^4 x}{2^x + 1} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{6}} \frac{\sin^4 x dx}{2^x + 1} + \int_0^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = \int_0^{\frac{\pi}{6}} \sin^4 x dx = \frac{1}{4} \int_0^{\frac{\pi}{6}} (1 - \cos 2x)^2 dx$$

$$= \frac{1}{8} \int_0^{\frac{\pi}{6}} (3 - 4 \cos 2x + \cos 4x) dx = \frac{4\pi - 7\sqrt{3}}{64}$$

$$7. I = \int_1^{e^\pi} \cos(\ln x) dx$$

$$\text{Đặt } t = \ln x \Rightarrow x = e^t \Rightarrow dx = e^t dt$$

$$\Rightarrow I = \int_0^\pi e^t \cos t dt = -\frac{1}{2}(e^\pi + 1) \text{ (dùng pp tích phân từng phần).}$$

$$8. I = \int_0^{\frac{\pi}{2}} e^{\sin^2 x} \cdot \sin x \cdot \cos^3 x dx$$

$$\text{Đặt } t = \sin^2 x \Rightarrow I = \frac{1}{2} \int_0^1 e^t (1-t) dt = \frac{1}{2} e \text{ (dùng tích phân từng phần)}$$

$$9. I = \int_0^{\frac{\pi}{4}} \ln(1 + \tan x) dx$$

$$\text{Đặt } t = \frac{\pi}{4} - x \Rightarrow I = \int_0^{\frac{\pi}{4}} \ln\left(1 + \tan\left(\frac{\pi}{4} - t\right)\right) dt = \int_0^{\frac{\pi}{4}} \ln\left(1 + \frac{1 - \tan t}{1 + \tan t}\right) dt = \int_0^{\frac{\pi}{4}} \ln \frac{2}{1 + \tan t} dt$$

$$= \int_0^{\frac{\pi}{4}} \ln 2 dt - \int_0^{\frac{\pi}{4}} \ln(1 + \tan t) dt = t \cdot \ln 2 \Big|_0^{\frac{\pi}{4}} - I$$

$$\Rightarrow 2I = \frac{\pi}{4} \ln 2 \Rightarrow I = \frac{\pi}{8} \ln 2.$$

$$10. I = \int_0^{\frac{\pi}{2}} \sin x \ln(1 + \sin x) dx$$

$$\text{Đặt } \begin{cases} u = \ln(1 + \sin x) \\ dv = \sin x dx \end{cases} \Rightarrow \begin{cases} du = \frac{1 + \cos x}{1 + \sin x} dx \\ v = -\cos x \end{cases}$$

$$\Rightarrow I = -\cos x \cdot \ln(1 + \sin x) \Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \cdot \frac{\cos x}{1 + \sin x} dx = 0 + \int_0^{\frac{\pi}{2}} \frac{1 - \sin^2 x}{1 + \sin x} dx = \int_0^{\frac{\pi}{2}} (1 - \sin x) dx = \frac{\pi}{2} - 1$$

$$11. I = \int_0^{\frac{\pi}{4}} \frac{\tan x \cdot \ln(\cos x)}{\cos x} dx$$

$$\text{Đặt } t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow I = -\int_1^{\frac{1}{\sqrt{2}}} \frac{\ln t}{t^2} dt = \int_{\frac{1}{\sqrt{2}}}^1 \frac{\ln t}{t^2} dt.$$

$$\text{Đặt } \begin{cases} u = \ln t \\ dv = \frac{1}{t^2} dt \end{cases} \Rightarrow \begin{cases} du = \frac{1}{t} dt \\ v = -\frac{1}{t} \end{cases} \Rightarrow I = \sqrt{2} - 1 - \frac{\sqrt{2}}{2} \ln 2$$

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PHẦN VII TUYỂN TẬP MỘT SỐ ĐỀ THI THỬ

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HT 1. Tính các tích phân sau:

1. $I = \int_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + \ln^2 x \right) dx$

2. $\int_1^e \frac{\ln(1+\ln^2 x)}{x} dx$

3. $\int_1^e \frac{\ln x - 2}{x \ln x + x} dx$

4. $I = \int_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + 3x^2 \ln x \right) dx$

5. $\int_1^e \frac{(x-2)\ln x + x}{x(1+\ln x)} dx$

6. $\int_{\frac{3\pi}{4}}^{\pi} \left[\frac{1}{x^2} + x \left(\frac{x}{\cos^2 x} + 2 \tan x \right) \right] dx$

7. $I = \int_{e^2}^{e^3} \frac{2x \ln x (\ln x - 1) + 3}{x(1 - \ln x)} dx$

8. $I = \int_1^2 \frac{x^3 \cdot 3^{x^2+1} + \ln(x+1)}{x^2} dx$

9. $\int_1^e \frac{\ln x}{x\sqrt{1+\ln x}} dx$

10. $\int_1^e \frac{x \ln x + \ln(xe^2)}{x \ln x + 1} dx$

11. $I = \int_1^e \frac{(x^3+1)\ln x + 2x^2+1}{2+x \ln x} dx$

12. $A = \int_0^{\frac{\pi}{2}} \sin x \cos x \ln(1+\sin^2 x) dx$

13. $\int_1^e \frac{\ln x - 2}{x \ln x + x} dx$

14. $I = \int_{\sqrt{2}}^e x^3 \ln \frac{x^2-1}{x^2+1} dx$

15. $\int_1^e \frac{x^2 \ln x + x \ln^2 x + x + 1}{x^2 + x \ln x} dx$

16. $I = \int_0^1 x \ln(x^2 + x + 1) dx$

17. $\int_{\frac{1}{2}}^2 \left(x + 1 - \frac{1}{x} \right) e^{x+\frac{1}{x}} dx$

Bài giải

1. $I = \int_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}} + \ln^2 x \right) dx$

$$I_1 = \int_1^e \frac{\ln x}{x\sqrt{1+\ln x}} dx, \text{ Đặt } t = \sqrt{1+\ln x}, \dots \text{ Tính được } I_1 = \frac{4}{3} - \frac{2\sqrt{2}}{3}$$

$$I_2 = \int_1^e (\ln^2 x) dx, \text{ lấy tích phân từng phần 2 lần được } I_2 = e - 2 \text{ Vậy: } I = I_1 + I_2 = e - \frac{2}{3} - \frac{2\sqrt{2}}{3}$$

$$2. \int_1^e \frac{\ln(1 + \ln^2 x)}{x} dx$$

Đặt $\ln x = t$, ta có $I = \int_0^1 \ln(1 + t^2) dt$. Đặt $u = \ln(1 + t^2)$, $dv = dt$ ta có: $du = \frac{2t}{1 + t^2} dt$, $v = t$.

$$\text{Từ đó có: } I = t \ln(1 + t^2) \Big|_0^1 - 2 \int_0^1 \frac{t^2}{1 + t^2} dt = \ln 2 - 2 \left(\int_0^1 dt - \int_0^1 \frac{dt}{1 + t^2} \right) (*)$$

Tiếp tục đặt $t = \tan u$, ta tính được $\int_0^1 \frac{dt}{1 + t^2} = \frac{\pi}{4}$. Thay vào (*) ta có: $I = \ln 2 - 2 + \frac{\pi}{2}$.

$$3. \int_1^e \frac{\ln x - 2}{x \ln x + x} dx = \int_1^e \frac{\ln x - 2}{(\ln x + 1)x} dx$$

Đặt $t = \ln x + 1 \Rightarrow dt = \frac{1}{x} dx$;

Đổi cận: $x = 1$ thì $t = 1$; $x = e$ thì $t = 2$

$$\text{Suy ra: } I = \int_1^2 \frac{t - 3}{t} dt = \int_1^2 \left(1 - \frac{3}{t} \right) dt = (t - \ln |t|) \Big|_1^2 = 1 - \ln 2$$

$$4. I = \int_1^e \left(\frac{\ln x}{x\sqrt{1 + \ln x}} + 3x^2 \ln x \right) dx$$

$$I = \int_1^e \frac{\ln x}{x\sqrt{1 + \ln x}} dx + 3 \int_1^e x^2 \ln x dx = I_1 + 3I_2$$

$$\text{+) Tính } I_1 = \int_1^e \frac{\ln x}{x\sqrt{1 + \ln x}} dx.$$

Đặt $t = \sqrt{1 + \ln x} \Rightarrow t^2 = 1 + \ln x$; $2t dt = \frac{1}{x} dx$

Khi $x = 1 \Rightarrow t = 1$; $x = e \Rightarrow t = \sqrt{2}$

$$\Rightarrow I_1 = \int_1^{\sqrt{2}} \frac{\sqrt{2}(t^2 - 1)}{t} \cdot 2t dt = 2 \int_1^{\sqrt{2}} (t^2 - 1) dt = 2 \left(\frac{t^3}{3} - t \right) \Big|_1^{\sqrt{2}} = \frac{2(2 - \sqrt{2})}{3}$$

$$+) \text{Tính } I_2 = \int_1^e x^2 \ln x dx. \text{ Đặt } \begin{cases} u = \ln x \\ dv = x^2 dx \end{cases} \Rightarrow \begin{cases} du = \frac{dx}{x} \\ v = \frac{x^3}{3} \end{cases}$$

$$\Rightarrow I_2 = \frac{x^3}{3} \cdot \ln x \Big|_1^e - \frac{1}{3} \int_1^e x^2 dx = \frac{e^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} \Big|_1^e = \frac{e^3}{3} - \frac{e^3}{9} + \frac{1}{9} = \frac{2e^3 + 1}{9}$$

$$I = I_1 + 3I_2 = \frac{5 - 2\sqrt{2} + 2e^3}{3}$$

$$5. \int_1^e \frac{(x-2)\ln x + x}{x(1+\ln x)} dx$$

$$I = \int_1^e \frac{x(1+\ln x) - 2\ln x}{x(1+\ln x)} dx = \int_1^e dx - 2 \int_1^e \frac{\ln x}{x(1+\ln x)} dx \text{ Ta có: } \int_1^e dx = e - 1$$

$$\text{Tính } J = \int_1^e \frac{\ln x}{x(1+\ln x)} dx$$

$$\text{Đặt } t = 1 + \ln x, \text{ Ta có: } J = \int_1^2 \frac{t-1}{t} dt = \int_1^2 \left(1 - \frac{1}{t}\right) dt = (t - \ln|t|) \Big|_1^2 = 1 - \ln 2$$

$$\text{Vậy } I = e - 1 - 2(1 - \ln 2) = e - 3 + 2\ln 2$$

$$6. \int_{\frac{3\pi}{4}}^{\pi} \left[\frac{1}{x^2} + x \left(\frac{x}{\cos^2 x} + 2 \tan x \right) \right] dx$$

$$\text{Ta có: } I = \int_{\frac{3\pi}{4}}^{\pi} \left[\frac{1}{x^2} + x \left(\frac{x}{\cos^2 x} + 2 \tan x \right) \right] dx = \int_{\frac{3\pi}{4}}^{\pi} \frac{1}{x^2} dx + \int_{\frac{3\pi}{4}}^{\pi} \frac{x^2}{\cos^2 x} dx + \int_{\frac{3\pi}{4}}^{\pi} 2x \tan x dx \quad (1)$$

$$+) \int_{\frac{3\pi}{4}}^{\pi} \frac{1}{x^2} dx = - \int_{\frac{3\pi}{4}}^{\pi} e^x d\left(\frac{1}{x}\right) = -e^x \Big|_{\frac{3\pi}{4}}^{\pi} = -e^{\pi} + e^{\frac{3\pi}{4}}$$

$$+) J = \int_{\frac{3\pi}{4}}^{\pi} \frac{x^2}{\cos^2 x} dx : \text{ Đặt } \begin{cases} u = x^2 \\ dv = \frac{1}{\cos^2 x} dx \end{cases} \Rightarrow \begin{cases} du = 2x dx \\ v = \tan x \end{cases} \Rightarrow J = \left(x^2 \tan x \right) \Big|_{\frac{3\pi}{4}}^{\pi} - \int_{\frac{3\pi}{4}}^{\pi} 2x \tan x dx$$

$$J = \frac{9\pi^2}{16} - \int_{\frac{3\pi}{4}}^{\pi} 2x \tan x dx$$

$$\text{Thay vào (1) ta có } I = -e^\pi + e^{3\pi} + \frac{9\pi^2}{16}$$

$$\begin{aligned} 7. I &= \int_{e^2}^{e^3} \frac{2x \ln x (\ln x - 1) + 3}{x(1 - \ln x)} dx = 3 \int_{e^2}^{e^3} \frac{1}{x(1 - \ln x)} dx - 2 \int_{e^2}^{e^3} \ln x dx = 3 \int_{e^2}^{e^3} \frac{1}{(1 - \ln x)} d(\ln x) - 2 \left(x \ln x \Big|_{e^2}^{e^3} - \int_{e^2}^{e^3} dx \right) \\ &= -3 \ln \left(|1 - \ln x| \right) \Big|_{e^2}^{e^3} - 2 \left(x \ln x \Big|_{e^2}^{e^3} - x \Big|_{e^2}^{e^3} \right) = -3 \ln 2 - 4e^3 + 2e^2. \end{aligned}$$

$$I = \int_1^e \frac{x^2 + x \ln x + 1}{x} e^x dx$$

$$I = \int_1^e \frac{x^2 + x \ln x + 1}{x} e^x dx = \int_1^e x e^x dx + \int_1^e \ln x e^x dx + \int_1^e \frac{e^x}{x} dx$$

$$\text{Đặt } I_1 = \int_1^e x e^x dx = x e^x \Big|_1^e - \int_1^e e^x dx = e^e (e - 1)$$

$$\text{Đặt } I_2 = \int_1^e e^x \ln x dx = e^x \ln x \Big|_1^e - \int_1^e \frac{e^x}{x} dx = e^e - \int_1^e \frac{e^x}{x} dx$$

$$\text{Vậy } I = I_1 + I_2 + \int_1^e \frac{e^x}{x} dx = e^{e+1} - e^e + e^e - \int_1^e \frac{e^x}{x} dx + \int_1^e \frac{e^x}{x} dx = e^{e+1}$$

$$8. I = \int_1^2 \frac{x^3 \cdot 3^{x^2+1} + \ln(x+1)}{x^2} dx$$

$$\text{Ta có } I = \int_1^2 x \cdot 3^{x^2+1} dx + \int_1^2 \frac{\ln(x+1)}{x^2} dx = J + K$$

$$\text{Tính: } J = \int_1^2 x \cdot 3^{x^2+1} dx = \frac{1}{2} \int_1^2 3^{x^2+1} d(x^2 + 1) = \frac{3^{x^2+1}}{2 \ln 3} \Big|_1^2 = \frac{117}{\ln 3}.$$

$$\text{Tính: } K = \int_1^2 \frac{\ln(x+1)}{x^2} dx. \text{ Đặt } \begin{cases} u = \ln(x+1) \\ v' = \frac{1}{x^2} \end{cases} \Rightarrow \begin{cases} u' = \frac{1}{x+1} \\ v = -\frac{1}{x} \end{cases}.$$

$$\text{Suy ra } K = -\frac{\ln(x+1)}{x} \Big|_1^2 + \int_1^2 \frac{dx}{x(x+1)} = -\frac{\ln 3}{2} + \ln 2 + \int_1^2 \left(\frac{1}{x} - \frac{1}{x+1} \right) dx = \frac{2 \ln 2 - \ln 3}{2} + \ln \left| \frac{x}{x+1} \right| \Big|_1^2$$

$$= \frac{2 \ln 2 - \ln 3}{2} + \ln \frac{2}{3} - \ln \frac{1}{2} = 3 \ln 2 - \frac{3}{2} \ln 3.$$

Vậy $I = \frac{117}{\ln 3} + 3 \ln 2 - \frac{3}{2} \ln 3.$

$$9. \int_1^e \frac{\ln x}{x \sqrt{1 + \ln x}} dx$$

Đặt $t = \sqrt{1 + \ln x}$ có $2t dt = \frac{1}{x} dx$ $x = 1$ thì $t = 1$; $x = e$ thì $t = \sqrt{2}$

$$\int_1^e \frac{\ln x}{x \sqrt{1 + \ln x}} dx = \int_1^{\sqrt{2}} \frac{t^2 - 1}{t} 2t dt = 2 \left(\frac{t^3}{3} - t \right) \Big|_1^{\sqrt{2}} = \frac{2(2 - \sqrt{2})}{3}$$

$$\int_1^e \frac{x \ln x + \ln(x.e^2)}{x \ln x + 1} dx$$

$$10. I = \int_1^e \frac{x \ln x + \ln(x.e^2)}{x \ln x + 1} dx.$$

$$I = \int_1^e \frac{x \ln x + 1 + \ln x + 1}{x \ln x + 1} dx = \int_1^e dx + \int_1^e \frac{\ln x + 1}{x \ln x + 1} dx = x \Big|_1^e + \int_1^e \frac{d(x \ln x + 1)}{x \ln x + 1}$$

$$= e - 1 + \ln |x \ln x + 1| \Big|_1^e = e - 1 + \ln(e + 1)$$

$$11. I = \int_1^e \frac{(x^3 + 1) \ln x + 2x^2 + 1}{2 + x \ln x} dx$$

$$I = \int_1^e \frac{(x^3 + 1) \ln x + 2x^2 + 1}{2 + x \ln x} dx = \int_1^e x^2 dx + \int_1^e \frac{1 + \ln x}{2 + x \ln x} dx$$

Ta có: $\int_1^e x^2 dx = \left(\frac{x^3}{3} \right) \Big|_1^e = \frac{e^3 - 1}{3}$

$$\int_1^e \frac{1 + \ln x}{2 + x \ln x} dx = \int_1^e \frac{d(2 + x \ln x)}{2 + x \ln x} = (\ln |2 + x \ln x|) \Big|_1^e = \ln(e + 2) - \ln 2 = \ln \frac{e + 2}{2}$$

Vậy $I = \frac{e^3 - 1}{3} + \ln \frac{e + 2}{2}.$

$$12. A = \int_0^{\frac{\pi}{2}} \sin x \cos x \ln(1 + \sin^2 x) dx$$

$$A = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin 2x \ln(1 + \sin^2 x) dx$$

Đặt $u = \ln(1 + \sin^2 x)$ và $dv = \sin 2x dx$. Suy ra: $du = \frac{\sin 2x}{1 + \sin^2 x} dx$ và $v = 1 + \sin^2 x$

$$A = \frac{1}{2} \left[(1 + \sin^2 x) \ln(1 + \sin^2 x) \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin 2x dx \right] = \frac{1}{2} \left[(1 + \sin^2 x) \ln(1 + \sin^2 x) \Big|_0^{\frac{\pi}{2}} - (\sin^2 x) \Big|_0^{\frac{\pi}{2}} \right] = \frac{\ln 4 - 1}{2}$$

$$13. \int_1^e \frac{\ln x - 2}{x \ln x + x} dx$$

$$\text{Ta có: } I = \int_1^e \frac{\ln x - 2}{x \ln x + x} dx = \int_1^e \frac{\ln x - 2}{(\ln x + 1)x} dx$$

Đặt $t = \ln x + 1 \Rightarrow dt = \frac{1}{x} dx$; Đổi cận: $x = 1$ thì $t = 1$; $x = e$ thì $t = 2$

$$\text{Suy ra: } I = \int_1^2 \frac{t-3}{t} dt = \int_1^2 \left(1 - \frac{3}{t} \right) dt = (t - \ln |t|) \Big|_1^2 = 1 - \ln 2$$

$$14. I = \int_{\sqrt{2}}^e x^3 \ln \frac{x^2 - 1}{x^2 + 1} dx$$

$$\text{Đặt } \begin{cases} u = \ln \frac{x^2 - 1}{x^2 + 1} \\ dv = x^3 dx \end{cases} \text{ ta có } \begin{cases} du = \frac{4x}{x^4 - 1} dx \\ v = \frac{x^4 - 1}{4} \end{cases}$$

$$I = \frac{x^4 - 1}{4} \ln \frac{x^2 - 1}{x^2 + 1} \Big|_{\sqrt{2}}^e - \int_{\sqrt{2}}^e x dx = \frac{e^4 - 1}{4} \ln \frac{e^2 - 1}{e^2 + 1} - \frac{x^2}{2} \Big|_{\sqrt{2}}^e = \frac{e^4 - 1}{4} \ln \frac{e^2 - 1}{e^2 + 1} + \frac{3}{4} \ln 3 - \frac{e^2}{2} + 1$$

$$15. \int_1^e \frac{x^2 \ln x + x \ln^2 x + x + 1}{x^2 + x \ln x} dx$$

$$I = \int_1^e \ln x dx + \int_1^e \frac{1 + \frac{1}{x}}{x + \ln x} dx = x(\ln x - 1) \Big|_1^e + \int_1^e \frac{d(x + \ln x)}{x + \ln x}$$

$$= [x(\ln x - 1) + \ln(x + \ln x)] \Big|_1^e = \ln[e(e + 1)].$$

$$16. I = \int_0^1 x \ln(x^2 + x + 1) dx$$

$$\text{Đặt } \begin{cases} u = \ln(x^2 + x + 1) \\ dv = x dx \end{cases} \Rightarrow \begin{cases} du = \frac{2x+1}{x^2+x+1} dx \\ v = \frac{x^2}{2} \end{cases}$$

$$I = \frac{x^2}{2} \ln(x^2 + x + 1) \Big|_0^1 - \frac{1}{2} \int_0^1 \frac{2x^3 + x^2}{x^2 + x + 1} dx = \frac{3}{4} \ln 3 - \frac{3}{4} J \quad \text{với}$$

$$J = \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}. \text{Đặt } x + \frac{1}{2} = \frac{\sqrt{3}}{2} \tan t, t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \Rightarrow J = \frac{2\sqrt{3}}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dt = \frac{\pi\sqrt{3}}{9}.$$

$$\text{Vậy } I = \frac{3}{4} \ln 3 - \frac{\pi\sqrt{3}}{12}$$

$$17. \int_{\frac{1}{2}}^2 \left(x + 1 - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$$

$$I = \int_{\frac{1}{2}}^2 \left(x + 1 - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx = \int_{\frac{1}{2}}^2 e^{x+\frac{1}{x}} dx + \int_{\frac{1}{2}}^2 \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx = I_1 + I_2.$$

$$\text{Tính } I_1 \text{ theo phương pháp từng phần } I_1 = \left. x e^{x+\frac{1}{x}} \right|_{\frac{1}{2}}^2 - \int_{\frac{1}{2}}^2 \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx = \frac{3}{2} e^{\frac{5}{2}} - I_2 \Rightarrow I = \frac{3}{2} e^{\frac{5}{2}}.$$

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HT 2. Tính các tích phân sau:

$$1. A = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{x\sqrt{1-x^2}}$$

$$2. I = \int_0^1 x^3 \ln \left(\frac{4-x^2}{4+x^2} \right) dx$$

$$3. \int_0^1 \frac{dx}{1+\sqrt{1-x^2}}$$

$$4. I = \int_2^6 \frac{dx}{2x+1+\sqrt{4x+1}}$$

$$5. I = \int_0^1 \frac{x^3+3x}{x^4-5x^2+6} dx$$

$$6. I = \int_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}} \frac{3x^4+x^2+1}{x^2\sqrt[3]{x^3+x}} dx$$

$$7. \int_{\frac{1}{3}}^1 \frac{x}{3x+\sqrt{9x^2-1}} dx$$

$$8. I = \int_{\sqrt{3}}^{2\sqrt{2}} \frac{x^4}{\left(x-\frac{1}{x}\right)\sqrt{x^2+1}} dx$$

$$9. I = \int_1^5 \frac{x^2+1}{x\sqrt{3x+1}} dx$$

$$10. \int_0^1 \frac{x(e^x+1)}{(x+1)^2} dx$$

$$11. \int_1^2 \frac{1-x^5}{x(1+x^5)^2} dx$$

$$12. \int_0^1 \left(x^2 e^{x^3} + \frac{\sqrt[4]{x}}{1+\sqrt{x}} \right) dx$$

Bài giải

$$1. A = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dx}{x\sqrt{1-x^2}}$$

$$\text{Đặt } t = \sqrt{1-x^2} \Rightarrow t^2 = 1-x^2 \Rightarrow 2tdt = -2xdx \Rightarrow \frac{dx}{x} = -\frac{tdt}{x^2}$$

$$\Rightarrow \frac{dx}{x} = -\frac{tdt}{1-t^2} = \frac{tdt}{t^2-1}$$

$$+ \text{Đổi cận: } x = \frac{1}{2} \Rightarrow t = \frac{\sqrt{3}}{2}; x = \frac{\sqrt{3}}{2} \Rightarrow t = \frac{1}{2}$$

$$A = \int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{tdt}{t^2-1} = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{dt}{1-t^2} = \frac{1}{2} \ln \left| \frac{t+1}{1-t} \right| \Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = \frac{1}{2} \ln \left(\frac{7+4\sqrt{3}}{3} \right)$$

$$2. I = \int_0^1 x^3 \ln \left(\frac{4-x^2}{4+x^2} \right) dx$$

$$\text{Đặt } \begin{cases} u = \ln \left(\frac{4-x^2}{4+x^2} \right) \\ dv = x^3 dx \end{cases} \Rightarrow \begin{cases} du = \frac{16x}{x^4-16} dx \\ v = \frac{x^4-16}{4} \end{cases}$$

$$\text{Do đó } I = \frac{1}{4} (x^4 - 16) \ln \left(\frac{4-x^2}{4+x^2} \right) \Big|_0^1 - 4 \int_0^1 x dx = -\frac{15}{4} \ln \left(\frac{3}{5} \right) - 2$$

$$3. \int_0^1 \frac{dx}{1 + \sqrt{1-x^2}}$$

Đặt $x = \sin t$ với $t \in [-\frac{\pi}{2}; \frac{\pi}{2}]$. Ta có: $dx = \cos t dt$ và $\sqrt{1-x^2} = \sqrt{1-\sin^2 t} = \sqrt{\cos^2 t} = |\cos t| = \cos t$.

Đổi cận: Với $x=0$ thì $t=0$; Với $x=1$ thì $t = \frac{\pi}{2}$. Từ đó:

$$\begin{aligned} \int_0^1 \frac{dx}{1 + \sqrt{1-x^2}} &= \int_0^{\pi/2} \frac{\cos t dt}{1 + \cos t} = \int_0^{\pi/2} \frac{2 \cos^2(t/2) - 1}{2 \cos^2(t/2)} dt \\ &= \int_0^{\pi/2} dt - \int_0^{\pi/2} \frac{d(t/2)}{\cos^2(t/2)} = (t - \tan(t/2)) \Big|_0^{\pi/2} = \frac{\pi}{2} - 1 \end{aligned}$$

$$4. I = \int_2^6 \frac{dx}{2x+1 + \sqrt{4x+1}}$$

$$\text{Đặt } t = \sqrt{4x+1} \Rightarrow x = \frac{t^2-1}{4} \Rightarrow dx = \frac{t dt}{2} \quad t(2) = 3, t(6) = 5$$

$$\text{Khi đó } I = \int_3^5 \frac{t dt}{(t+1)^2} = \int_3^5 \left(\frac{1}{t+1} - \frac{1}{(t+1)^2} \right) dt = \left(\ln|t+1| + \frac{1}{t+1} \right) \Big|_3^5 = \ln \frac{3}{2} - \frac{1}{12}$$

$$= \left(\ln|t+1| + \frac{1}{t+1} \right) \Big|_3^5 = \ln \frac{3}{2} - \frac{1}{12}$$

$$5. I = \int_0^1 \frac{x^3 + 3x}{x^4 - 5x^2 + 6} dx$$

$$I = \frac{1}{2} \int_0^1 \frac{x^2 + 3}{(x^2-2)(x^2-3)} dx^2 = \frac{1}{2} \int_0^1 \frac{x^2 - 2 + 5}{(x^2-2)(x^2-3)} dx^2$$

$$= \frac{1}{2} \int_0^1 \frac{dx^2}{x^2 - 3} + \frac{5}{2} \int_0^1 \left(\frac{1}{x^2 - 3} - \frac{1}{x^2 - 2} \right) dx^2 = \left(\frac{1}{2} \ln |x^2 - 3| + \frac{5}{2} \ln \left| \frac{x^2 - 3}{x^2 - 2} \right| \right) \Big|_0^1$$

$$= \left(\frac{1}{2} \ln 2 + \frac{5}{2} \ln 2 \right) - \left(\frac{1}{2} \ln 3 + \frac{5}{2} \ln \frac{3}{2} \right) = 3 \ln 2 - 3 \ln 3 + \frac{5}{2} \ln 2 = 3 \ln \frac{2}{3} + \frac{5}{2} \ln 2$$

$$6. I = \int \frac{\frac{1}{\sqrt{7}}}{\frac{1}{\sqrt{26}}} \frac{3x^4 + x^2 + 1}{x^2 \sqrt[3]{x^3 + x}} dx = \int \frac{\frac{1}{\sqrt{7}}}{\frac{1}{\sqrt{26}}} \frac{3x^4 + x^2}{x^2 \sqrt[3]{x^3 + x}} dx + \int \frac{\frac{1}{\sqrt{7}}}{\frac{1}{\sqrt{26}}} \frac{1}{x^2 \sqrt[3]{x^3 + x}} dx = I_1 + I_2$$

$$* \text{ Tính } I_2 = \int \frac{\frac{1}{\sqrt{7}}}{\frac{1}{\sqrt{26}}} \frac{1}{\sqrt[3]{1 + \frac{1}{x^2}}} \frac{1}{x^3} dx = -\frac{1}{2} \int \frac{\frac{1}{\sqrt{7}}}{\frac{1}{\sqrt{26}}} \frac{1}{\sqrt[3]{1 + \frac{1}{x^2}}} d \left(1 + \frac{1}{x^2} \right) = -\frac{3}{4} \sqrt[3]{\left(1 + \frac{1}{x^2} \right)^2} \Big|_{\frac{1}{\sqrt{26}}}^{\frac{1}{\sqrt{7}}} = \frac{15}{4}$$

$$\text{Vậy: } I = \frac{322}{91}.$$

$$7. \int_{\frac{1}{3}}^1 \frac{x}{3x + \sqrt{9x^2 - 1}} dx$$

$$I = \int_{\frac{1}{3}}^1 \frac{x}{3x + \sqrt{9x^2 - 1}} dx = \int_{\frac{1}{3}}^1 x(3x - \sqrt{9x^2 - 1}) dx = \int_{\frac{1}{3}}^1 3x^2 dx - \int_{\frac{1}{3}}^1 x\sqrt{9x^2 - 1} dx$$

$$I_1 = \int_{\frac{1}{3}}^1 3x^2 dx = x^3 \Big|_{\frac{1}{3}}^1 = \frac{26}{27}$$

$$I_2 = \int_{\frac{1}{3}}^1 x\sqrt{9x^2 - 1} dx = \frac{1}{18} \int_{\frac{1}{3}}^1 \sqrt{9x^2 - 1} d(9x^2 - 1) = \frac{1}{27} (9x^2 - 1)^{\frac{3}{2}} \Big|_{\frac{1}{3}}^1 = \frac{16\sqrt{2}}{27}.$$

$$\text{Vậy } I = \frac{26 - 16\sqrt{2}}{27}$$

$$8. I = \int_{\frac{1}{\sqrt{3}}}^{2\sqrt{2}} \frac{x^4}{\left(x - \frac{1}{x}\right)\sqrt{x^2 + 1}} dx$$

$$\text{Ta có: } I = \int_{\frac{1}{\sqrt{3}}}^{2\sqrt{2}} \frac{x^5}{(x^2 - 1)\sqrt{x^2 + 1}} dx$$

Đặt $t = \sqrt{x^2 + 1}$, suy ra $dt = \frac{x}{\sqrt{x^2 + 1}} dx$ & $x^2 = t^2 - 1$ Đổi cận: $x = \sqrt{3} \Rightarrow t = 2$; $x = 2\sqrt{2} \Rightarrow t = 3$

$$\text{Khi đó } I = \int_2^3 \frac{(t^2 - 1)^2}{t^2 - 2} dt \text{ Ta có } I = \int_2^3 \frac{t^4 - 2t^2 + 1}{t^2 - 2} dt = \int_2^3 t^2 dt + \int_2^3 \frac{1}{t^2 - 2} dt = \frac{1}{3} t^3 \Big|_2^3 + \frac{1}{2\sqrt{2}} \int_2^3 \left(\frac{1}{t - \sqrt{2}} - \frac{1}{t + \sqrt{2}} \right) dt$$

$$= \frac{19}{3} + \frac{1}{2\sqrt{2}} \left[\ln|t - \sqrt{2}| - \ln|t + \sqrt{2}| \right]_2^3 = \frac{19}{3} + \frac{\sqrt{2}}{4} \ln \left(\frac{4 + \sqrt{2}}{4 - \sqrt{2}} \right)$$

$$9. I = \int_1^5 \frac{x^2 + 1}{x\sqrt{3x + 1}} dx$$

Đặt $t = \sqrt{3x + 1} \Rightarrow dt = \frac{3dx}{2\sqrt{3x + 1}} \Rightarrow dx = \frac{2tdt}{3}$.

Khi $x = 1$ thì $t = 2$, và khi $x = 5$ thì $t = 4$.

$$\text{Suy ra } I = \int_2^4 \frac{\left(\frac{t^2 - 1}{3} \right)^2 + 1}{\frac{t^2 - 1}{3} \cdot t} \cdot \frac{2tdt}{3} = \frac{2}{9} \int_2^4 (t^2 - 1) dt + 2 \int_2^4 \frac{dt}{t^2 - 1} = \frac{2}{9} \left(\frac{1}{3} t^3 - t \right) \Big|_2^4 + \ln \left| \frac{t - 1}{t + 1} \right| \Big|_2^4 = \frac{100}{27} + \ln \frac{9}{5}.$$

$$10. \int_0^1 \frac{x(e^x + 1)}{(x + 1)^2} dx$$

$$\text{Đặt } \begin{cases} u = x(e^x + 1) \\ dv = \frac{dx}{(x + 1)^2} \end{cases} \Rightarrow \begin{cases} du = (e^x(x + 1) + 1)dx \\ v = \frac{-1}{x + 1} \end{cases}$$

$$I = -\frac{x(e^x + 1)}{x + 1} \Big|_0^1 + \int_0^1 \left(e^x + \frac{1}{x + 1} \right) dx = -\frac{e + 1}{2} + (e^x + \ln|x + 1|) \Big|_0^1 = \frac{e}{2} + \ln 2 - \frac{3}{2}.$$

$$\text{Vậy } I = \frac{e}{2} + \ln 2 - \frac{3}{2}$$

$$11. \int_1^2 \frac{1 - x^5}{x(1 + x^5)^2} dx$$

$$\int_1^2 \frac{1 - x^5}{x(1 + x^5)^2} dx = \int_1^2 \frac{1 + x^5 - 2x^5}{x(1 + x^5)^2} dx = \int_1^2 \frac{1}{x(1 + x^5)} dx - \int_1^2 \frac{2x^4}{(1 + x^5)^2} dx = I_1 - I_2$$

$$x = \frac{1}{t} \Rightarrow I_1 = \int_{\frac{1}{2}}^{\frac{1}{2}} \frac{\left(-\frac{1}{t^2}\right)}{\frac{1}{t}\left(1 + \frac{1}{t^5}\right)} dt = \int_{\frac{1}{2}}^1 \frac{t^4}{t^5 + 1} dt = \frac{1}{5} \int_{\frac{1}{2}}^1 \frac{1}{t^5 + 1} d(t^5 + 1) = \frac{1}{5} \ln|t^5 + 1| \Big|_{\frac{1}{2}}^1 = \frac{1}{5} (6 \ln 2 - \ln 33)$$

$$I_2 = \frac{2}{5} \int_1^2 \frac{1}{(1+x^5)^2} d(x^5 + 1) = -\frac{2}{5} \cdot \left. \frac{1}{x^5 + 1} \right|_1^2 = \frac{31}{165}$$

$$I = \frac{1}{5} (6 \ln 2 - \ln 33) - \frac{31}{165}$$

$$12. \int_0^1 \left(x^2 e^{x^3} + \frac{\sqrt[4]{x}}{1 + \sqrt{x}} \right) dx$$

$$\text{Đặt } I = \int_0^1 \left(x^2 e^{x^3} + \frac{\sqrt[4]{x}}{1 + \sqrt{x}} \right) dx. \text{ Ta có } I = \int_0^1 x^2 e^{x^3} dx + \int_0^1 \frac{\sqrt[4]{x}}{1 + \sqrt{x}} dx$$

$$\text{Ta tính } I_1 = \int_0^1 x^2 e^{x^3} dx \text{ Đặt } t = x^3 \text{ ta có } I_1 = \frac{1}{3} \int_0^1 e^t dt = \frac{1}{3} e^t \Big|_0^1 = \frac{1}{3} e - \frac{1}{3}$$

$$\text{Ta tính } I_2 = \int_0^1 \frac{\sqrt[4]{x}}{1 + \sqrt{x}} dx \text{ Đặt } t = \sqrt[4]{x} \Rightarrow x = t^4 \Rightarrow dx = 4t^3 dt$$

$$\text{Khi đó } I_2 = 4 \int_0^1 \frac{t^4}{1 + t^2} dx = 4 \int_0^1 \left(t^2 - 1 + \frac{1}{1 + t^2} \right) dt = 4 \left(-\frac{2}{3} + \frac{\pi}{4} \right). \quad \text{Vậy } I = I_1 + I_2 = \frac{1}{3} e + \pi - 3$$

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HT 3. Tính các tích phân sau:

$$1. \int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx$$

$$2. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x + 2 \sin x - 3) \cos x}{\sin^3 x} dx$$

$$3. M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx$$

$$4. I = \int_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x} dx$$

$$5. \int_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3} \sin 2x + 2 \cos^2 x} dx$$

$$6. I = \int_0^{\frac{\pi}{4}} \frac{x \sin x + \sin 2x}{\cos^2 x} dx$$

$$7. M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx$$

$$8. I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos \left(2x - \frac{\pi}{4} \right)} dx$$

$$9. I = \int_0^{\frac{\pi}{2}} \cos 2x (\sin^4 x + \cos^4 x) dx$$

$$10. I = \int_0^{\frac{\pi}{4}} \frac{\tan x}{(4 \cos x - \sin x) \cos x} dx$$

$$11. \int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx$$

$$12. \int_0^{\frac{\pi}{2}} \frac{x}{1 + \sin x} dx$$

$$13. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x + 2 \sin x - 3) \cos x}{\sin^3 x} dx$$

$$14. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot x dx}{1 + \sin^4 x}$$

$$15. \int_0^{\frac{\pi}{3}} \frac{3 \sin x - \sin 2x}{(\cos 2x - 3 \cos x + 1)(3 - 2 \sin^2 x)} dx$$

$$16. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x \cdot \sin \left(x + \frac{\pi}{4} \right)} dx$$

$$17. I = \int_0^{\frac{\pi}{2}} x (\cos x + \sin^5 x) dx$$

Bài giải

$$1. I = \int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx = 2 \int_0^{\frac{\pi}{2}} e^{\cos x} \cdot \cos x \cdot \sin x dx + \int_0^{\frac{\pi}{2}} \sin x \cdot \sin 2x dx$$

$$J = \int_0^{\frac{\pi}{2}} e^{\cos x} \cdot \cos x \cdot \sin x dx$$

$$\text{Đặt } t = \cos x \text{ có } J = \int_0^1 t \cdot e^t \cdot dt = t \cdot e^t \Big|_0^1 - \int_0^1 e^t \cdot dt = 1$$

$$K = \int_0^{\frac{\pi}{2}} \sin x \cdot \sin 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx = \frac{1}{2} (\sin x - \frac{1}{3} \sin 3x) \Big|_0^{\frac{\pi}{2}} = \frac{2}{3}$$

$$\text{Vậy: } I = \int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx = 2 + \frac{2}{3} = \frac{8}{3}$$

$$2. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x + 2 \sin x - 3) \cos x}{\sin^3 x} dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x + 2 \sin x - 3) \cos x}{\sin^3 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^3 x} dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(2 \sin x - 3) \cos x}{\sin^3 x} dx$$

$$I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^3 x} dx = -\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x d\left(\frac{1}{\sin^2 x}\right) = -\frac{1}{2} \frac{x}{\sin^2 x} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^2 x} dx = -\frac{1}{2} \left(\frac{\pi}{2} - \frac{\pi}{4}\right) - \frac{1}{2} \cot x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2}$$

$$I_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(2 \sin x - 3) \cos x}{\sin^3 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{2 \sin x - 3}{\sin^3 x} d(\sin x) = 2\sqrt{2} - \frac{7}{2}. \quad \text{Vậy } I = I_1 + I_2 = 2\sqrt{2} - 3.$$

$$3. M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx$$

$$M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx = \underbrace{\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1 + \cos 2x} dx}_{M_1} + \underbrace{\int_0^{\frac{\pi}{4}} \frac{\cos x}{1 + \cos 2x} dx}_{M_2};$$

$$M_1 = -\frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{d(1 + \cos 2x)}{1 + \cos 2x} = -\frac{1}{2} \ln |1 + \cos 2x| \Big|_0^{\frac{\pi}{4}} = \frac{1}{2} \ln 2, \quad M_2 = \int_0^{\frac{\pi}{4}} \frac{\cos x}{1 + \cos 2x} dx = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{\cos x}{1 - \sin^2 x} dx =$$

$$\text{Đặt } u = \sin t \quad M_2 = \frac{1}{2} \int_0^{\frac{1}{\sqrt{2}}} \frac{du}{1 - u^2} = \frac{1}{4} \int_0^{\frac{1}{\sqrt{2}}} \left(\frac{1}{1 - u} + \frac{1}{1 + u} \right) du = \frac{1}{4} \ln \left| \frac{u + 1}{u - 1} \right| \Big|_0^{\frac{1}{\sqrt{2}}} = \frac{1}{2} \ln(1 + \sqrt{2})$$

$$\text{Vậy } M = \frac{1}{2} \ln(2 + 2\sqrt{2})$$

$$4. I = \int_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x} dx$$

$$I = \int_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x} dx = - \int_0^{\frac{\pi}{6}} \frac{\tan^2 x + 1}{(t \operatorname{an} x + 1)^2} dx$$

$$\text{Đặt } t = t \operatorname{an} x \Rightarrow dt = \frac{1}{\cos^2 x} dx = (\tan^2 x + 1) dx$$

$$\text{Đổi cận: } x = 0 \Rightarrow t = 0, x = \frac{\pi}{6} \Rightarrow t = \frac{1}{\sqrt{3}}$$

$$\text{Suy ra } I = - \int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(t+1)^2} = \frac{1}{t+1} \Big|_0^{\frac{1}{\sqrt{3}}} = \frac{1-\sqrt{3}}{2}$$

$$5. \int_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3} \sin 2x + 2 \cos^2 x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sqrt{3} \sin 2x + 2 \cos^2 x} dx = \int_0^{\frac{\pi}{2}} \sqrt{(\sin x - \sqrt{3} \cos x)^2} dx = \int_0^{\frac{\pi}{2}} |\sin x - \sqrt{3} \cos x| dx$$

$$\sin x - \sqrt{3} \cos x = 0 \Leftrightarrow \tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} + k\pi \quad \text{Do } x \in \left(0; \frac{\pi}{2}\right) \text{ nên } x = \frac{\pi}{3}$$

$$I = \int_0^{\frac{\pi}{3}} |\sin x - \sqrt{3} \cos x| dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} |\sin x - \sqrt{3} \cos x| dx = \left| \int_0^{\frac{\pi}{3}} (\sin x - \sqrt{3} \cos x) dx \right| + \left| \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sin x - \sqrt{3} \cos x) dx \right|$$

$$= \left| \left(-\cos x - \sqrt{3} \sin x \right) \Big|_0^{\frac{\pi}{3}} \right| + \left| \left(-\cos x - \sqrt{3} \sin x \right) \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} \right| = \left| -\frac{1}{2} - \frac{3}{2} + 1 \right| + \left| -\sqrt{3} + \frac{1}{2} + \frac{3}{2} \right| = 3 - \sqrt{3}$$

$$6. I = \int_0^{\frac{\pi}{4}} \frac{x \sin x + \sin 2x}{\cos^2 x} dx$$

$$+ \text{Ta có } I = \int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^2 x} dx + 2 \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx \quad \text{Đặt } I_1 = \int_0^{\frac{\pi}{4}} \frac{x \sin x}{\cos^2 x} dx; I_2 = 2 \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx$$

+ Tính I_1 : Đặt $u = x \Rightarrow du = dx; v = \int \frac{\sin x}{\cos^2 x} dx = -\int \cos^{-2} x d(\cos x) = \frac{1}{\cos x}$

$$\Rightarrow I_1 = \frac{x}{\cos x} \Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \frac{x}{\cos x} \Big|_0^{\frac{\pi}{4}} - \frac{1}{2} \ln \left| \frac{1 + \sin x}{1 - \sin x} \right| \Big|_0^{\frac{\pi}{4}} = \frac{\pi\sqrt{2}}{4} - \frac{1}{2} \ln \frac{2 + \sqrt{2}}{2 - \sqrt{2}}$$

+ Tính $I_2 = -2 \int_0^{\frac{\pi}{4}} \frac{d(\cos x)}{\cos x} = -2 \ln |\cos x| \Big|_0^{\frac{\pi}{4}} = -2 \ln \frac{\sqrt{2}}{2}$

Vậy $I = I_1 + I_2 = \frac{\pi\sqrt{2}}{4} - \frac{1}{2} \ln \frac{2 + \sqrt{2}}{2 - \sqrt{2}} - 2 \ln \frac{\sqrt{2}}{2}$

7. $M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx$

$$M = \int_0^{\frac{\pi}{4}} \frac{\cos x + \sin 2x}{1 + \cos 2x} dx = \underbrace{\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{1 + \cos 2x} dx}_{M_1} + \underbrace{\int_0^{\frac{\pi}{4}} \frac{\cos x}{1 + \cos 2x} dx}_{M_2}$$

$$M_1 = -\frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{d(1 + \cos 2x)}{1 + \cos 2x} = -\frac{1}{2} \ln |1 + \cos 2x| \Big|_0^{\frac{\pi}{4}} = \frac{1}{2} \ln 2 \quad M_2 = \int_0^{\frac{\pi}{4}} \frac{\cos x}{1 + \cos 2x} dx = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{\cos x}{1 - \sin^2 x} dx =$$

Đặt $u = \sin t \quad M_2 = \frac{1}{2} \int_0^{\frac{\sqrt{2}}{2}} \frac{du}{1 - u^2} = \frac{1}{4} \int_0^{\frac{\sqrt{2}}{2}} \left(\frac{1}{1 - u} + \frac{1}{1 + u} \right) du = \frac{1}{4} \ln \left| \frac{u + 1}{u - 1} \right| \Big|_0^{\frac{\sqrt{2}}{2}} = \frac{1}{2} \ln(1 + \sqrt{2})$

Vậy $M = \frac{1}{2} \ln(2 + 2\sqrt{2})$

8. $I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos \left(2x - \frac{\pi}{4} \right)} dx$

$$I = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos \left(2x - \frac{\pi}{4} \right)} dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot x - \tan x}{\sin 2x \cos \left(2x - \frac{\pi}{4} \right)} dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\sin x \cos x \left(\cos 2x \cos \frac{\pi}{4} + \sin 2x \sin \frac{\pi}{4} \right)} dx$$

$$= 2\sqrt{2} \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot 2x}{\sin 2x (\cos 2x + \sin 2x)} dx = 2\sqrt{2} \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\cot 2x}{1 + \cot 2x} \cdot \frac{1}{\sin^2 2x} dx$$

Đặt $t = \cot 2x \Rightarrow dt = -\frac{2}{\sin^2 2x} dx \Rightarrow -\frac{1}{2} dt = \frac{1}{\sin^2 2x} dx$. Đổi cận: $x = \frac{\pi}{8} \Rightarrow t = 1$; $x = \frac{\pi}{4} \Rightarrow t = 0$

$$I = 2\sqrt{2} \int_1^0 \frac{t}{1+t} \cdot \left(-\frac{1}{2} dt\right) = \sqrt{2} \int_0^1 \frac{t}{1+t} dt = \sqrt{2} \int_0^1 \left(1 - \frac{1}{t+1}\right) dt = \sqrt{2} (t - \ln|t+1|) \Big|_0^1 = \sqrt{2} (1 - \ln 2)$$

$$9. I = \int_0^{\frac{\pi}{2}} \cos 2x (\sin^4 x + \cos^4 x) dx$$

$$I = \int_0^{\frac{\pi}{2}} \cos 2x \left(1 - \frac{1}{2} \sin^2 2x\right) dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2} \sin^2 2x\right) d(\sin 2x)$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} d(\sin 2x) - \frac{1}{4} \int_0^{\frac{\pi}{2}} \sin^2 2x d(\sin 2x) = \frac{1}{2} \sin 2x \Big|_0^{\frac{\pi}{2}} - \frac{1}{12} \sin^3 2x \Big|_0^{\frac{\pi}{2}} = 0$$

$$10. I = \int_0^{\frac{\pi}{4}} \frac{\tan x}{(4 \cos x - \sin x) \cos x} dx$$

$$\text{Ta có: } I = \int_0^{\frac{\pi}{4}} \frac{\tan x}{(4 - \tan x) \cos^2 x} dx$$

Đặt: $\tan x - 4 = t \Rightarrow \frac{dx}{\cos^2 x} = dt$. Đổi cận: Với $x = 0 \Rightarrow t = -4$; $x = \frac{\pi}{4} \Rightarrow t = -3$

$$\text{Suy ra: } I = -\int_{-4}^{-3} \frac{(t+4) \cdot dt}{t} = -\int_{-4}^{-3} \left(1 + \frac{4}{t}\right) dt = -(t + 4 \ln|t|) \Big|_{-4}^{-3} = 4 \ln \frac{4}{3} - 1$$

$$11. \int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx$$

$$\int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx = 2 \int_0^{\frac{\pi}{2}} e^{\cos x} \cdot \cos x \cdot \sin x dx + \int_0^{\frac{\pi}{2}} \sin x \cdot \sin 2x dx$$

$$I = \int_0^{\frac{\pi}{2}} e^{\cos x} \cdot \cos x \cdot \sin x dx$$

$$\text{Đặt } t = \cos x \text{ có } I = \int_0^1 t \cdot e^t \cdot dt = t \cdot e^t \Big|_0^1 - \int_0^1 e^t \cdot dt = 1$$

$$K = \int_0^{\frac{\pi}{2}} \sin x \cdot \sin 2x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx = \frac{1}{2} (\sin x - \frac{1}{3} \sin 3x) \Big|_0^{\frac{\pi}{2}} = \frac{2}{3}$$

$$\int_0^{\frac{\pi}{2}} (e^{\cos x} + \sin x) \cdot \sin 2x dx = 2 + \frac{2}{3} = \frac{8}{3}$$

$$12. \int_0^{\pi} \frac{x}{1 + \sin x} dx$$

$$I = \int_0^{\pi} \frac{x}{1 + \sin x} dx = \int_0^{\pi} \frac{x}{(\sin \frac{x}{2} + \cos \frac{x}{2})^2} dx = \int_0^{\pi} \frac{x}{2 \cos^2 \left(\frac{x}{2} - \frac{\pi}{4} \right)} dx$$

$$\text{Đặt } \begin{cases} u = x \\ dv = \frac{dx}{2 \cos^2 \left(\frac{x}{2} - \frac{\pi}{4} \right)} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \tan \left(\frac{x}{2} - \frac{\pi}{4} \right) \end{cases} \Rightarrow I = x \tan \left(\frac{x}{2} - \frac{\pi}{4} \right) \Big|_0^{\pi} - \int_0^{\pi} \tan \left(\frac{x}{2} - \frac{\pi}{4} \right) dx$$

$$\Rightarrow I = \pi + 2 \ln \left| \cos \left(\frac{x}{2} - \frac{\pi}{4} \right) \right| \Big|_0^{\pi}$$

$$13. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x + 2 \sin x - 3) \cos x}{\sin^3 x} dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(x + 2 \sin x - 3) \cos x}{\sin^3 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^3 x} dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(2 \sin x - 3) \cos x}{\sin^3 x} dx$$

$$I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^3 x} dx = -\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x d \left(\frac{1}{\sin^2 x} \right) = -\frac{1}{2} \frac{x}{\sin^2 x} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^2 x} dx = -\frac{1}{2} \left(\frac{\pi}{2} - \frac{\pi}{2} \right) - \frac{1}{2} \cot x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2}$$

$$I_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(2 \sin x - 3) \cos x}{\sin^3 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{2 \sin x - 3}{\sin^3 x} d(\sin x) = 2\sqrt{2} - \frac{7}{2} \quad \text{Vậy } I = I_1 + I_2 = 2\sqrt{2} - 3.$$

$$14. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot x dx}{1 + \sin^4 x}$$

$$\text{Ta có: } I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot x}{1 + \sin^4 x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x(1 + \sin^4 x)} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin^3 x \cos x}{\sin^4 x(1 + \sin^4 x)} dx.$$

$$\text{Đặt } t = \sin^4 x, \text{ ta có: } x = \frac{\pi}{4} \Rightarrow t = \frac{1}{4}, x = \frac{\pi}{2} \Rightarrow t = 1 \text{ và } dt = 4 \sin^3 x \cos x dx.$$

$$\text{Khi đó } I = \frac{1}{4} \int_{\frac{1}{4}}^1 \frac{dt}{t(t+1)} = \frac{1}{4} \int_{\frac{1}{4}}^1 \left(\frac{1}{t} - \frac{1}{t+1} \right) dt = \frac{1}{4} \ln \frac{t}{t+1} \Big|_{\frac{1}{4}}^1 = \frac{1}{4} \ln \frac{5}{2}.$$

$$15. \int_0^{\frac{\pi}{3}} \frac{3 \sin x - \sin 2x}{(\cos 2x - 3 \cos x + 1)(3 - 2 \sin^2 x)} dx$$

$$\text{Ta có } I = \int_0^{\frac{\pi}{3}} \frac{3 \sin x - \sin 2x}{(2 \cos^2 x - 3 \cos x)(3 - 2 \sin^2 x)} dx = \int_0^{\frac{\pi}{3}} \frac{\sin x(3 - 2 \cos x)}{(2 \cos x - 3) \cdot \cos x \cdot (1 + 2 \cos^2 x)} dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sin x(3 - 2 \cos x)}{(2 \cos x - 3) \cdot \cos x \cdot (1 + 2 \cos^2 x)} dx = \int_0^{\frac{\pi}{3}} \frac{-\sin x}{\cos x \cdot (1 + 2 \cos^2 x)} dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\cos x \cdot (-\sin x) dx}{\cos^2 x \cdot (1 + 2 \cos^2 x)}. \text{ Đặt } t = 2 \cos^2 x \Rightarrow dt = 4 \cos x \cdot (-\sin x) dx$$

$$\text{Đổi cận: Khi } x = 0 \Rightarrow t = 2; \text{ khi } x = \frac{\pi}{3} \Rightarrow t = \frac{1}{2}. \text{ Khi đó } I = \frac{1}{2} \int_2^{\frac{1}{2}} \frac{dt}{t(1+t)} =$$

$$= \frac{1}{2} \int_2^{\frac{1}{2}} \left(\frac{1}{t} - \frac{1}{t+1} \right) dt = \frac{1}{2} \ln \left| \frac{t}{t+1} \right| \Big|_2^{\frac{1}{2}} = \frac{1}{2} \cdot \left(\ln \frac{1}{3} - \ln \frac{2}{3} \right) = \frac{1}{2} \cdot \ln \frac{1}{2}. \text{ Vậy } I = -\frac{1}{2} \ln 2.$$

$$16. I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x \cdot \sin\left(x + \frac{\pi}{4}\right)} dx$$

$$\text{Tính } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x \sin\left(x + \frac{\pi}{4}\right)} dx = \sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x (\sin x + \cos x)} dx = \sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin^2 x (1 + \cot x)} dx$$

$$\text{Đặt } 1 + \cot x = t \Rightarrow \frac{1}{\sin^2 x} dx = -dt \quad \text{Khi } x = \frac{\pi}{6} \Leftrightarrow t = 1 + \sqrt{3}; \quad x = \frac{\pi}{3} \Leftrightarrow t = \frac{\sqrt{3} + 1}{\sqrt{3}}$$

$$\text{Vậy } I = \sqrt{2} \int_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} \frac{t-1}{t} dt = \sqrt{2} (t - \ln t) \Big|_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} = \sqrt{2} \left(\frac{2}{\sqrt{3}} - \ln \sqrt{3} \right)$$

$$17. I = \int_0^{\pi} x (\cos x + \sin^5 x) dx$$

$$I = \int_0^{\pi} x (\cos x + \sin^5 x) dx$$

$$* I = \int_0^{\pi} x (\cos x + \sin^5 x) dx = \underbrace{\int_0^{\pi} x \cos x dx}_{I_1} + \underbrace{\int_0^{\pi} x \sin^5 x dx}_{I_2} .$$

$$* I_1 = \int_0^{\pi} x \cos x dx = x \sin x \Big|_0^{\pi} - \int_0^{\pi} \sin x dx = x \sin x \Big|_0^{\pi} + \cos x \Big|_0^{\pi} = -2$$

$$* \text{ Với } I_2 \text{ ta đặt } x = \pi - t \Rightarrow I_2 = \frac{\pi}{2} \int_0^{\pi} (1 - \cos^2 x)^2 d(-\cos x) = \frac{8\pi}{15} . \quad * \text{ Vậy } I = \frac{8\pi}{15} - 2$$

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HT 4. Tính các tích phân sau:

$$1. I = \int_0^{\ln 6} \frac{e^x}{3\sqrt{3+e^x+2e^x+7}} dx.$$

$$2. I = \int_0^1 \frac{(x-1)e^x + x + 1}{1+e^x} dx$$

$$3. I = \int_0^1 \frac{(x^2+x)e^x}{x+e^{-x}} dx$$

$$4. I = \int_0^{\ln 2} \frac{x}{e^x + e^{-x} + 2} dx.$$

Bài giải

$$1. I = \int_0^{\ln 6} \frac{e^x}{3\sqrt{3+e^x+2e^x+7}} dx.$$

Đặt $\sqrt{3+e^x} = t$. Khi đó $e^x = t^2 - 3 \Rightarrow e^x dx = 2t dt$.

Khi $x = 0 \Rightarrow t = 2$, khi $x = \ln 6 \Rightarrow t = 3$.

$$\text{Suy ra } I = \int_2^3 \frac{2t dt}{3t + 2(t^2 - 3) + 7} = 2 \int_2^3 \frac{t}{2t^2 + 3t + 1} dt$$

$$= 2 \int_2^3 \frac{t}{(t+1)(2t+1)} dt = 2 \int_2^3 \left(\frac{1}{t+1} - \frac{1}{2t+1} \right) dt = 2 \ln |t+1| \Big|_2^3 - \ln |2t+1| \Big|_2^3 = (2 \ln 4 - 2 \ln 3) - (\ln 7 - \ln 5) = \ln \frac{80}{63}.$$

$$2. I = \int_0^1 \frac{(x-1)e^x + x + 1}{1+e^x} dx$$

$$I = \int_0^1 \frac{xe^x - e^x + x + 1}{1+e^x} dx = \int_0^1 \frac{x(e^x + 1) + (1+e^x) - 2e^x}{1+e^x} dx = \int_0^1 (x+1) dx - 2 \int_0^1 \frac{e^x}{1+e^x} dx = I_1 - 2I_2$$

$$\text{Tính } I_1 = \int_0^1 (x+1) dx = \left(\frac{x^2}{2} + x \right) \Big|_0^1 = \frac{3}{2} \quad \text{Tính } I_2 = \int_0^1 \frac{e^x}{1+e^x} dx = \int_0^1 \frac{d(e^x+1)}{e^x+1} = \ln(e^x+1) \Big|_0^1 = \ln \frac{e+1}{2}$$

$$\text{Vậy } I = \frac{3}{2} - 2 \ln \frac{e+1}{2}.$$

$$3. I = \int_0^1 \frac{(x^2+x)e^x}{x+e^{-x}} dx$$

$$\text{Ta có } I = \int_0^1 \frac{(x^2+x)e^x}{x+e^{-x}} dx = \int_0^1 \frac{xe^x \cdot (x+1)e^x}{xe^x+1} dx$$

$$\text{Đặt } t = xe^x + 1 \Rightarrow dt = (x+1)e^x dx \quad x=0 \Rightarrow t=1; x=1 \Rightarrow t=e+1$$

$$\text{Suy ra } I = \int_0^1 \frac{x e^x \cdot (x+1) e^x}{x e^x + 1} dx = \int_1^{e+1} \frac{(t-1)}{t} dt = \int_1^{e+1} \left(1 - \frac{1}{t}\right) dt. \text{ Vậy } I = \left(t - \ln|t|\right) \Big|_1^{e+1} = e - \ln(e+1).$$

$$4. I = \int_0^{\ln 2} \frac{x}{e^x + e^{-x} + 2} dx.$$

$$\text{Ta có } I = \int_0^{\ln 2} \frac{x e^x}{(e^x + 1)^2} dx.$$

$$\text{Đặt } u = x \Rightarrow du = dx, \quad dv = \frac{e^x}{(e^x + 1)^2} dx \Rightarrow v = -\frac{1}{e^x + 1}.$$

$$\text{Theo công thức tích phân từng phần ta có: } I = -\frac{x}{e^x + 1} \Big|_0^{\ln 2} + \int_0^{\ln 2} \frac{dx}{e^x + 1} = -\frac{\ln 2}{3} + \int_0^{\ln 2} \frac{dx}{e^x + 1} \quad (1)$$

$$\text{Tính } I_1 = \int_0^{\ln 2} \frac{dx}{e^x + 1}. \text{ Đặt } e^x = t \text{ ta có } x = 0 \Rightarrow t = 1; x = \ln 2 \Rightarrow t = 2 \text{ và } dx = \frac{dt}{t}.$$

$$\text{Suy ra } I_1 = \int_1^2 \frac{dt}{t(t+1)} = \int_1^2 \left(\frac{1}{t} - \frac{1}{t+1}\right) dt = \ln t \Big|_1^2 - \ln(t+1) \Big|_1^2 = 2 \ln 2 - \ln 3.$$

$$\text{Thay vào (1) ta được } I = \frac{5}{3} \ln 2 - \ln 3.$$

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PHẦN VIII TÍCH PHÂN HÀM TRỊ TUYỆT ĐỐI

ỨNG DỤNG TÍCH PHÂN

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HT 1. Tính các tích phân sau:

$$1. I = \int_{-3}^2 |x^2 - 3x + 2| dx.$$

$$2. I = \int_0^{\frac{\pi}{2}} \sqrt{5 - 4 \cos^2 x - 4 \sin x} dx$$

$$3. I = \int_{-1}^2 (|x| - |x-1|) dx$$

Bài giải

$$1. I = \int_{-3}^2 |x^2 - 3x + 2| dx$$

Bảng xét dấu

x	-3	1	2
$x^2 - 3x + 2$	+	0	-

$$I = \int_{-3}^1 (x^2 - 3x + 2) dx - \int_1^2 (x^2 - 3x + 2) dx = \frac{59}{2}.$$

$$2. I = \int_0^{\frac{\pi}{2}} \sqrt{5 - 4 \cos^2 x - 4 \sin x} dx.$$

$$I = \int_0^{\frac{\pi}{2}} \sqrt{4 \sin^2 x - 4 \sin x + 1} dx = \int_0^{\frac{\pi}{2}} |2 \sin x - 1| dx.$$

Bảng xét dấu

x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$
$2 \sin x - 1$	-	0	+

$$I = -\int_0^{\frac{\pi}{6}} (2 \sin x - 1) dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 \sin x - 1) dx = 2\sqrt{3} - 2 - \frac{\pi}{6}.$$

$$3. I = \int_{-1}^2 (|x| - |x-1|) dx.$$

Cách 1.

$$\begin{aligned} I &= \int_{-1}^2 (|x| - |x-1|) dx = \int_{-1}^2 |x| dx - \int_{-1}^2 |x-1| dx \\ &= -\int_{-1}^0 x dx + \int_0^2 x dx + \int_{-1}^1 (x-1) dx - \int_1^2 (x-1) dx \\ &= -\frac{x^2}{2} \Big|_{-1}^0 + \frac{x^2}{2} \Big|_0^2 + \left(\frac{x^2}{2} - x \right) \Big|_{-1}^1 - \left(\frac{x^2}{2} - x \right) \Big|_1^2 = 0. \end{aligned}$$

Cách 2.

Bảng xét dấu

x	-1	0	1	2
x	-	0	+	+
x-1	-	-	0	+

$$\begin{aligned} I &= \int_{-1}^0 (-x + x - 1) dx + \int_0^1 (x + x - 1) dx + \int_1^2 (x - x + 1) dx \\ &= -x \Big|_{-1}^0 + (x^2 - x) \Big|_0^1 + x \Big|_1^2 = 0. \end{aligned}$$

Vậy $I = 0$.

HT 2. Tính diện tích hình phẳng giới hạn bởi $y = \ln x$, $x = 1$, $x = e$ và Ox .

Bài giải

Do $\ln x \geq 0 \forall x \in [1; e]$ nên:

$$S = \int_1^e |\ln x| dx = \int_1^e \ln x dx = x(\ln x - 1) \Big|_1^e = 1.$$

Vậy $S = 1$ (đvdt).

HT 3. Tính diện tích hình phẳng giới hạn bởi $y = -x^2 + 4x - 3$, $x = 0$, $x = 3$ và Ox

Bài giải

Bảng xét dấu

x	0	1	3
y	-	0	+ 0

$$S = -\int_0^1 (-x^2 + 4x - 3) dx + \int_1^3 (-x^2 + 4x - 3) dx$$

$$= -\left(-\frac{x^3}{3} + 2x^2 + 3x\right)\Big|_0^1 + \left(-\frac{x^3}{3} + 2x^2 + 3x\right)\Big|_1^3 = \frac{8}{3}.$$

Vậy $S = \frac{8}{3}$ (đvdt).

HT 4. Tính diện tích hình phẳng giới hạn bởi các đường: $y = x^3 + 11x - 6$, $y = 6x^2$, $x = 0$, $x = 2$

Bài giải

$$\text{Đặt } h(x) = (x^3 + 11x - 6) - 6x^2 = x^3 - 6x^2 + 11x - 6$$

$$h(x) = 0 \Leftrightarrow x = 1 \vee x = 2 \vee x = 3 \text{ (loại)}.$$

Bảng xét dấu

x	0	1	2
h(x)	-	0	+ 0

$$S = -\int_0^1 (x^3 - 6x^2 + 11x - 6) dx + \int_1^2 (x^3 - 6x^2 + 11x - 6) dx$$

$$= -\left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right)\Big|_0^1 + \left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right)\Big|_1^2 = \frac{5}{2}.$$

Vậy $S = \frac{5}{2}$ (đvdt).

HT 5. Tính diện tích hình phẳng giới hạn bởi các đường: $y = x^3 + 11x - 6$, $y = 6x^2$

Bài giải

$$\text{Đặt } h(x) = (x^3 + 11x - 6) - 6x^2 = x^3 - 6x^2 + 11x - 6$$

$$h(x) = 0 \Leftrightarrow x = 1 \vee x = 2 \vee x = 3.$$

Bảng xét dấu

x	1	2	3
h(x)	0	+	0 - 0

$$S = \int_1^2 (x^3 - 6x^2 + 11x - 6) dx - \int_2^3 (x^3 - 6x^2 + 11x - 6) dx$$

$$= \left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x \right) \Big|_1^2 - \left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x \right) \Big|_2^3 = \frac{1}{2}.$$

Vậy $S = \frac{1}{2}$ (đvdt).

HT 6. Tính diện tích hình phẳng giới hạn bởi $y = x^3$, $y = 4x$.

Bài giải

Phương trình hoành độ giao điểm:

$$x^3 = 4x \Leftrightarrow x = -2 \vee x = 0 \vee x = 2$$

$$\Rightarrow S = \left| \int_{-2}^0 (x^3 - 4x) dx \right| + \left| \int_0^2 (x^3 - 4x) dx \right| = \left| \left(\frac{x^4}{4} - 2x^2 \right) \Big|_{-2}^0 \right| + \left| \left(\frac{x^4}{4} - 2x^2 \right) \Big|_0^2 \right| = 8.$$

Vậy $S = 8$ (đvdt).

HT 7. Tính diện tích hình phẳng giới hạn bởi $y = x^2 - 4|x| + 3$ và trục hoành.

Bài giải

Phương trình hoành độ giao điểm:

$$x^2 - 4|x| + 3 = 0 \Leftrightarrow t^2 - 4t + 3 = 0, t = |x| \geq 0 \Leftrightarrow \begin{cases} t = 1 \\ t = 3 \end{cases} \Leftrightarrow \begin{cases} |x| = 1 \\ |x| = 3 \end{cases} \Leftrightarrow \begin{cases} x = \pm 1 \\ x = \pm 3 \end{cases}$$

$$\Rightarrow S = \int_{-3}^3 |x^2 - 4|x| + 3| dx = 2 \int_0^3 |x^2 - 4x + 3| dx$$

$$= 2 \left[\int_0^1 (x^2 - 4x + 3) dx + \int_1^3 (x^2 - 4x + 3) dx \right]$$

$$= 2 \left[\left| \left(\frac{x^3}{3} - 2x^2 + 3x \right) \Big|_0^1 \right| + \left| \left(\frac{x^3}{3} - 2x^2 + 3x \right) \Big|_1^3 \right| = \frac{16}{3}.$$

Vậy $S = \frac{16}{3}$ (đvdt).

HT 8. Tính diện tích hình phẳng giới hạn bởi $y = |x^2 - 4x + 3|$ và $y = x + 3$.

Bài giải

Phương trình hoành độ giao điểm:

$$|x^2 - 4x + 3| = x + 3 \Leftrightarrow \begin{cases} x + 3 \geq 0 \\ x^2 - 4x + 3 = x + 3 \\ x^2 - 4x + 3 = -x - 3 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 5 \end{cases}.$$

Bảng xét dấu

x	0	1	3	5
$x^2 - 4x + 3$	+	0	-	0
	+	0	-	0
	+	0	-	+

$$\begin{aligned} \Rightarrow S &= \left| \int_0^1 (x^2 - 5x) dx + \int_1^3 (-x^2 + 3x - 6) dx + \int_3^5 (x^2 - 5x) dx \right| \\ &= \left| \left(\frac{x^3}{3} - \frac{5x^2}{2} \right) \Big|_0^1 + \left(-\frac{x^3}{3} + \frac{3x^2}{2} - 6x \right) \Big|_1^3 + \left(\frac{x^3}{3} - \frac{5x^2}{2} \right) \Big|_3^5 \right| = \frac{109}{6}. \end{aligned}$$

Vậy $S = \frac{109}{6}$ (đvdt).

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HT 9. Tính diện tích hình phẳng giới hạn bởi $y = |x^2 - 1|$, $y = |x| + 5$.

Bài giải

Phương trình hoành độ giao điểm:

$$|x^2 - 1| = |x| + 5 \Leftrightarrow |t^2 - 1| = t + 5, \quad t = |x| \geq 0$$

$$\Leftrightarrow \begin{cases} t = |x| \geq 0 \\ t^2 - 1 = t + 5 \\ t^2 - 1 = -t - 5 \end{cases} \Leftrightarrow \begin{cases} t = |x| \geq 0 \\ t = 3 \end{cases} \Leftrightarrow x = \pm 3$$

$$\Rightarrow S = \int_{-3}^3 \left| x^2 - 1 \right| - (|x| + 5) dx = 2 \int_0^3 \left| x^2 - 1 \right| - (x + 5) dx$$

Bảng xét dấu

x	0	1	3
$x^2 - 1$	-	0	+

$$\begin{aligned} \Rightarrow S &= 2 \left| \int_0^1 (-x^2 - x - 4) dx + \int_1^3 (x^2 - x - 6) dx \right| \\ &= 2 \left| \left(-\frac{x^3}{3} - \frac{x^2}{2} - 4x \right) \Big|_0^1 + \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x \right) \Big|_1^3 \right| = \frac{73}{3}. \end{aligned}$$

Vậy $S = \frac{73}{3}$ (đvdt).

HT 10. Tính diện tích hình phẳng giới hạn bởi $y = x$, $y = 0$, $y = \sqrt{2 - x^2}$.

Bài giải

Ta có: $y = \sqrt{2 - x^2} \Leftrightarrow x = \sqrt{2 - y^2}$, $x \geq 0$.

Phương trình tung độ giao điểm: $y = \sqrt{2 - y^2} \Leftrightarrow y = 1$.

$$\begin{aligned} \Rightarrow S &= \int_0^1 \left| \sqrt{2 - y^2} - y \right| dy = \left| \int_0^1 (\sqrt{2 - y^2} - y) dy \right| \\ &= \left| \int_0^{\frac{\pi}{4}} 2 \cos^2 t dt - \int_0^1 y dy \right| = \left| \left(t + \frac{1}{2} \sin 2t \right) \Big|_0^{\frac{\pi}{4}} - \frac{y^2}{2} \Big|_0^1 \right|. \end{aligned}$$

Vậy $S = \frac{\pi}{4}$ (đvdt).

HT 11. Tính thể tích hình cầu do hình tròn $(C): x^2 + y^2 = R^2$ quay quanh Ox.

Bài Giải

Hoành độ giao điểm của (C) và Ox là $x^2 = R^2 \Leftrightarrow x = \pm R$.

Phương trình (C): $x^2 + y^2 = R^2 \Leftrightarrow y^2 = R^2 - x^2$

$$\Rightarrow V = \pi \int_{-R}^R (R^2 - x^2) dx = 2\pi \int_0^R (R^2 - x^2) dx = 2\pi \left(R^2 x - \frac{x^3}{3} \right) \Big|_0^R = \frac{4\pi R^3}{3}.$$

Vậy $V = \frac{4\pi R^3}{3}$ (đvtt).

HT 12. Tính thể tích hình khối do ellipse (E): $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ quay quanh Oy.

Bài Giải

Tung độ giao điểm của (E) và Oy là $\frac{y^2}{b^2} = 1 \Leftrightarrow y = \pm b$.

Phương trình (E): $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Leftrightarrow x^2 = a^2 - \frac{a^2 y^2}{b^2}$

$$\Rightarrow V = \pi \int_{-b}^b \left(a^2 - \frac{a^2 y^2}{b^2} \right) dy = 2\pi \int_0^b \left(a^2 - \frac{a^2 y^2}{b^2} \right) dy = 2\pi \left(a^2 y - \frac{a^2 y^3}{3b^2} \right) \Big|_0^b = \frac{4\pi a^2 b}{3}.$$

Vậy $V = \frac{4\pi a^2 b}{3}$ (đvtt).

HT 13. Tính thể tích hình khối do hình phẳng giới hạn bởi các đường $y = x^2, y^2 = x$ quay quanh Ox.

Bài Giải

Hoàn độ giao điểm: $\begin{cases} x \geq 0 \\ x^4 = x \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 1 \end{cases}.$

$$\Rightarrow V = \pi \int_0^1 |x^4 - x| dx = \pi \int_0^1 (x^4 - x) dx = \pi \left(\frac{1}{5} x^5 - \frac{1}{2} x^2 \right) \Big|_0^1 = \frac{3\pi}{10}.$$

Vậy $V = \frac{3\pi}{10}$ (đvtt).

HT 14. Tính thể tích hình khối do hình phẳng giới hạn bởi các đường $x = -y^2 + 5, x = 3 - y$ quay quanh Oy.

Bài Giải

Tung độ giao điểm: $-y^2 + 5 = 3 - y \Leftrightarrow \begin{cases} y = -1 \\ y = 2 \end{cases}.$

$$\Rightarrow V = \pi \int_{-1}^2 \left| (-y^2 + 5)^2 - (3 - y)^2 \right| dy = \pi \int_{-1}^2 (y^4 - 11y^2 + 6y + 16) dy$$

$$= \pi \left| \left(\frac{y^5}{5} - \frac{11y^3}{3} + 3y^2 + 16y \right) \right|_{-1}^2 = \frac{153\pi}{5}.$$

Vậy $V = \frac{153\pi}{5}$ (đvtt).

HT 15. Tính diện tích hình phẳng giới hạn bởi các đường có phương trình sau

1) $y = \sin x, y = 0, x = 0, x = 2\pi$

2) $y = x^3, y = 0, x = -1, x = 2$

3) $y = x^2 - 2x, y = -x^2 + 4x$

4) $y = x^3, y = 4x, x = -1, x = 2$

5) $y = -x^2 - 5, y = -6x, x = 0, x = 1$

6) $y = -x^2 - 2, y = -3x, x = 0, x = 2$

7) $y = -x^2 - 2x, y = -x - 2$

8) $y = x^3 - 2x^2 - x + 2$ và trục hoành

9) $y = |x^3 - 2x^2 - x| + 2$ và trục hoành

10) $y = \sqrt{4 - \frac{x^2}{4}}, y = \frac{x^2}{4\sqrt{2}}$

11) $y = -\sqrt{4 - x^2}, x^2 + 3y = 0$

12) $y = |x^2 - 4x + 3|, y = 3$

13) $y = |x^2 - 4|x| + 3|, y = 0$

14) $x = y, x = \frac{\sqrt{3}}{\sqrt{4 - y^2}}$

15) $x = \frac{2}{y^2}, x = \frac{1}{\sqrt{8 - y^2}}, y = \sqrt{2} (y \geq 0)$

16) $y = (2 + \cos x) \sin x, y = 0, x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

17) $y = x\sqrt{1 + x^2}, y = 0, x = 1$

18) $y = \frac{\ln x}{2\sqrt{x}}, y = 0, x = 1, x = e$

19) $y = \frac{\sqrt{1 + \ln x}}{x}, y = 0, x = 1, x = e$

20) $y = 0, y = \ln x, x = 2, x = e$

21) $y = \frac{1}{\sin^2 x}, y = \frac{1}{\cos^2 x}, x = \frac{\pi}{6}, x = \frac{\pi}{3}$

22) $y = x^2, y = 4x^2, y = 4$

23) $y = x(x + 1)(x - 2), y = 0, x = -2, x = 2$

24) $y = xe^x, y = 0, x = -1, x = 2$

25) $y^2 = 4x, x - y + 1 = 0, y = 0$

26) $x - y^3 + 1 = 0, x + y - 1 = 0, y = 0$

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Bài giải

1) $S = \int_0^{2\pi} |\sin x| dx = \left| \int_0^{\pi} \sin x dx \right| + \left| \int_{\pi}^{2\pi} \sin x dx \right| = \left| -\cos x \Big|_0^{\pi} \right| + \left| -\cos x \Big|_{\pi}^{2\pi} \right| = 4$ (đvdt).

2) $S = \int_{-1}^2 |x^3| dx = \left| \int_{-1}^0 x^3 dx \right| + \left| \int_0^2 x^3 dx \right| = \left| \frac{x^4}{4} \Big|_{-1}^0 \right| + \left| \frac{x^4}{4} \Big|_0^2 \right| = \frac{17}{4}$ (đvdt).

3) $x^2 - 2x = -x^2 + 4x \Leftrightarrow x = 0 \vee x = 3$

$\Rightarrow S = \int_0^3 |(x^2 - 2x) - (-x^2 + 4x)| dx = \left| \int_0^3 (2x^2 - 6x) dx \right| = \left| \left(\frac{2x^3}{3} - 3x^2 \right) \Big|_0^3 \right| = 9$ (đvdt).

4) $x^3 - 4x = 0 \Leftrightarrow x = 0 \vee x = 2 \vee x = -2$ (loại).

$$\Rightarrow S = \int_{-1}^2 |x^3 - 4x| dx = \left| \int_{-1}^0 (x^3 - 4x) dx \right| + \left| \int_0^2 (x^3 - 4x) dx \right| = \left| \left(\frac{x^4}{4} - 2x^2 \right) \Big|_{-1}^0 \right| + \left| \left(\frac{x^4}{4} - 2x^2 \right) \Big|_0^2 \right|.$$

Vậy $S = \frac{23}{4}$ (đvdt).

5) $x^2 - 6x + 5 = 0 \Leftrightarrow x = 1 \vee x = 5$ (loại).

$$\Rightarrow S = \int_0^1 |x^2 - 6x + 5| dx = \left| \int_0^1 (x^2 - 6x + 5) dx \right| = \left| \left(\frac{x^3}{3} - 3x^2 + 5x \right) \Big|_0^1 \right|.$$

Vậy $S = \frac{7}{3}$ (đvdt).

6) $x^2 - 3x + 2 = 0 \Leftrightarrow x = 1 \vee x = 2$.

$$\begin{aligned} \Rightarrow S &= \int_0^2 |x^2 - 3x + 2| dx = \left| \int_0^1 (x^2 - 3x + 2) dx \right| + \left| \int_1^2 (x^2 - 3x + 2) dx \right| \\ &= \left| \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right) \Big|_0^1 \right| + \left| \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right) \Big|_1^2 \right| = 1 \text{ (đvdt)}. \end{aligned}$$

7) $-x^2 - 2x = -x - 2 \Leftrightarrow x = -2 \vee x = 1$.

$$\Rightarrow S = \int_{-2}^1 |x^2 + x - 2| dx = \left| \int_{-2}^1 (x^2 + x - 2) dx \right| = \left| \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x \right) \Big|_{-2}^1 \right|.$$

Vậy $S = \frac{9}{2}$ (đvdt).

8) $x^3 - 2x^2 - x + 2 = 0 \Leftrightarrow x = 2 \vee x = \pm 1$.

$$\begin{aligned} \Rightarrow S &= \int_{-1}^2 |x^3 - 2x^2 - x + 2| dx = \left| \int_{-1}^1 (x^3 - 2x^2 - x + 2) dx \right| + \left| \int_1^2 (x^3 - 2x^2 - x + 2) dx \right| \\ &= \left| \left(\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x \right) \Big|_{-1}^1 \right| + \left| \left(\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x \right) \Big|_1^2 \right|. \end{aligned}$$

Vậy $S = \frac{37}{12}$ (đvdt).

$$9) |x|^3 - 2x^2 - |x| + 2 = 0 \Leftrightarrow \begin{cases} t = |x| \geq 0 \\ t^3 - 2t^2 - t + 2 = 0 \end{cases} \Leftrightarrow \begin{cases} t = |x| \geq 0 \\ t = 1 \\ t = 2 \end{cases} \Leftrightarrow \begin{cases} x = \pm 1 \\ x = \pm 2 \end{cases}.$$

$$\begin{aligned} \Rightarrow S &= \int_{-2}^2 \left| |x|^3 - 2x^2 - |x| + 2 \right| dx = 2 \int_0^2 \left| x^3 - 2x^2 - x + 2 \right| dx \\ &= 2 \left| \int_0^1 (x^3 - 2x^2 - x + 2) dx \right| + 2 \left| \int_1^2 (x^3 - 2x^2 - x + 2) dx \right| \\ &= 2 \left| \left(\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x \right) \Big|_0^1 \right| + 2 \left| \left(\frac{x^4}{4} - \frac{2x^3}{3} - \frac{x^2}{2} + 2x \right) \Big|_1^2 \right| = 3(\text{đvdt}). \end{aligned}$$

$$10) \sqrt{4 - \frac{x^2}{4}} = \frac{x^2}{4\sqrt{2}} \Leftrightarrow x^4 + 8x^2 - 128 = 0 \Leftrightarrow x = \pm 2\sqrt{2}$$

$$\begin{aligned} \Rightarrow S &= \int_{-2\sqrt{2}}^{2\sqrt{2}} \left| \sqrt{4 - \frac{x^2}{4}} - \frac{x^2}{4\sqrt{2}} \right| dx = \left| \int_{-2\sqrt{2}}^{2\sqrt{2}} \left(\sqrt{4 - \frac{x^2}{4}} - \frac{x^2}{4\sqrt{2}} \right) dx \right| \\ &= 2 \left| \int_0^{2\sqrt{2}} \left(\sqrt{4 - \frac{x^2}{4}} - \frac{x^2}{4\sqrt{2}} \right) dx \right| = \left| \int_0^{2\sqrt{2}} \sqrt{16 - x^2} dx - \frac{1}{2\sqrt{2}} \int_0^{2\sqrt{2}} x^2 dx \right| \\ &= \left| 16 \int_0^{\frac{\pi}{4}} \cos^2 t dt - \frac{1}{2\sqrt{2}} \int_0^{2\sqrt{2}} x^2 dx \right| = \left| 8 \left(t + \frac{1}{2} \sin 2t \right) \Big|_0^{\frac{\pi}{4}} - \frac{1}{2\sqrt{2}} \frac{x^3}{3} \Big|_0^{2\sqrt{2}} \right|. \end{aligned}$$

$$\text{Vậy } S = 2\pi + \frac{4}{3} (\text{đvdt}).$$

$$11) x^2 + 3y = 0 \Leftrightarrow y = -\frac{x^2}{3} \Rightarrow -\sqrt{4 - x^2} = -\frac{x^2}{3} \Leftrightarrow x^4 + 9x^2 - 36 = 0 \Leftrightarrow x = \pm\sqrt{3}$$

$$\begin{aligned} \Rightarrow S &= \int_{-\sqrt{3}}^{\sqrt{3}} \left| \sqrt{4 - x^2} - \frac{x^2}{3} \right| dx = 2 \left| \int_0^{\sqrt{3}} \left(\sqrt{4 - x^2} - \frac{x^2}{3} \right) dx \right| \\ &= 2 \left| \int_0^{\sqrt{3}} \sqrt{4 - x^2} dx - \frac{1}{3} \int_0^{\sqrt{3}} x^2 dx \right| = 2 \left| 4 \int_0^{\frac{\pi}{3}} \cos^2 t dt - \frac{1}{3} \int_0^{\sqrt{3}} x^2 dx \right| = 2 \left| 2 \left(t + \frac{1}{2} \sin 2t \right) \Big|_0^{\frac{\pi}{3}} - \frac{x^3}{9} \Big|_0^{\sqrt{3}} \right|. \end{aligned}$$

$$\text{Vậy } S = \frac{4\pi + \sqrt{3}}{3} \text{ (đvdt).}$$

$$12) |x^2 - 4x + 3| = 3 \Leftrightarrow \begin{cases} x^2 - 4x + 3 = 3 \\ x^2 - 4x + 3 = -3 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = 4 \end{cases}.$$

Bảng xét dấu

x	0	1	3	4
$x^2 - 4x + 3$	+	0	-	0

$$\begin{aligned} \Rightarrow S &= \int_0^4 |x^2 - 4x + 3| - 3 dx = \left| \int_0^1 (x^2 - 4x) dx \right| + \left| \int_1^3 (-x^2 + 4x - 6) dx \right| + \left| \int_3^4 (x^2 - 4x) dx \right| \\ &= \left| \left(\frac{x^3}{3} - 2x^2 \right) \Big|_0^1 \right| + \left| \left(-\frac{x^3}{3} + 2x^2 - 6x \right) \Big|_1^3 \right| + \left| \left(\frac{x^3}{3} - 2x^2 \right) \Big|_3^4 \right| = 8 \text{ (đvdt)}. \end{aligned}$$

$$13) |x^2 - 4|x| + 3| = 0 \Leftrightarrow x^2 - 4|x| + 3 = 0 \Leftrightarrow \begin{cases} |x| = 1 \\ |x| = 3 \end{cases} \Leftrightarrow \begin{cases} x = \pm 1 \\ x = \pm 3 \end{cases}.$$

Bảng xét dấu

x	0	1	3
$x^2 - 4 x + 3$	+	0	-

$$\begin{aligned} \Rightarrow S &= \int_{-3}^3 |x^2 - 4|x| + 3| dx = 2 \int_0^3 |x^2 - 4x + 3| dx \\ &= 2 \left[\int_0^1 (x^2 - 4x + 3) dx - \int_1^3 (x^2 - 4x + 3) dx \right] \\ &= 2 \left[\left(\frac{x^3}{3} - 2x^2 + 3x \right) \Big|_0^1 - \left(\frac{x^3}{3} - 2x^2 + 3x \right) \Big|_1^3 \right]. \end{aligned}$$

$$\text{Vậy } S = \frac{16}{3} \text{ (đvdt).}$$

$$14) \text{ Tung độ giao điểm } y = \frac{\sqrt{3}}{\sqrt{4-y^2}}, 0 \leq y < 2 \Leftrightarrow \begin{cases} y = 1 \\ y = \sqrt{3} \end{cases}$$

$$\Rightarrow S = \int_1^{\sqrt{3}} \left| \frac{\sqrt{3}}{\sqrt{4-y^2}} - y \right| dy = \int_1^{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{4-y^2}} - y \right) dy = \dots$$

$$\text{Vậy } S = 1 - \frac{\pi\sqrt{3}}{6} \text{ (đvdt).}$$

$$15) \text{ Tung độ giao điểm } \frac{2}{y^2} = \frac{1}{\sqrt{8-y^2}} \Leftrightarrow y = 2$$

$$\Rightarrow S = \int_{\sqrt{2}}^2 \left| \frac{2}{y^2} - \frac{1}{\sqrt{8-y^2}} \right| dy = \int_{\sqrt{2}}^2 \left(\frac{2}{y^2} - \frac{1}{\sqrt{8-y^2}} \right) dy = \dots$$

$$\text{Vậy } S = \sqrt{2} - 1 - \frac{\pi}{12} \text{ (đvdt).}$$

$$16) S = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} |(2 + \cos x) \sin x| dx = \int_{\frac{\pi}{2}}^{\pi} (2 + \cos x) \sin x dx - \int_{\pi}^{\frac{3\pi}{2}} (2 + \cos x) \sin x dx$$

$$= - \left(2 \cos x + \frac{1}{4} \cos 2x \right) \Big|_{\frac{\pi}{2}}^{\pi} + \left(2 \cos x + \frac{1}{4} \cos 2x \right) \Big|_{\pi}^{\frac{3\pi}{2}} = 3 \text{ (đvdt).}$$

$$17) \text{ Hoành độ giao điểm } x\sqrt{1+x^2} = 0 \Leftrightarrow x = 0$$

$$\Rightarrow S = \int_0^1 |x\sqrt{1+x^2}| dx = \int_0^1 x\sqrt{1+x^2} dx = \frac{1}{2} \int_0^1 \sqrt{1+x^2} d(1+x^2) = \frac{1}{3} \sqrt{(1+x^2)^3} \Big|_0^1.$$

$$\text{Vậy } S = \frac{2\sqrt{2}-1}{3} \text{ (đvdt).}$$

$$18) S = \int_1^e \left| \frac{\ln x}{2\sqrt{x}} \right| dx = \int_1^e \frac{\ln x}{2\sqrt{x}} dx \left(\frac{\ln x}{2\sqrt{x}} > 0 \forall x \in [1; e] \right).$$

$$\text{Đặt } t = \ln x \Rightarrow x = e^t \Rightarrow dx = e^t dt$$

$$x = 1 \Rightarrow t = 0, x = e \Rightarrow t = 1$$

$$\Rightarrow S = \int_0^1 \frac{te^t dt}{2\sqrt{e^t}} = \int_0^1 t d(\sqrt{e^t}) = t\sqrt{e^t} \Big|_0^1 - \int_0^1 \sqrt{e^t} dt = \sqrt{e} - 2\sqrt{e^t} \Big|_0^1.$$

$$\text{Vậy } S = 2 - \sqrt{e} \text{ (đvdt).}$$

$$19) S = \int_1^e \left| \frac{\sqrt{1 + \ln x}}{x} \right| dx = \int_1^e \frac{\sqrt{1 + \ln x}}{x} dx.$$

$$\text{Đặt } t = \sqrt{1 + \ln x} \Rightarrow t^2 = 1 + \ln x \Rightarrow 2t dt = \frac{dx}{x}$$

$$x = 1 \Rightarrow t = 1, \quad x = e \Rightarrow t = \sqrt{2}$$

$$\Rightarrow S = \int_1^{\sqrt{2}} t \cdot 2t dt = \int_1^{\sqrt{2}} 2t^2 dt = \frac{2}{3} t^3 \Big|_1^{\sqrt{2}}.$$

$$\text{Vậy } S = \frac{4\sqrt{2} - 2}{3} \text{ (đvdt).}$$

$$20) S = \int_2^e |\ln x| dx = \int_2^e \ln x dx = x \ln x \Big|_2^e - \int_2^e dx.$$

$$\text{Vậy } S = 2 - 2 \ln 2.$$

$$21) \frac{1}{\cos^2 x} = \frac{1}{\sin^2 x} \Leftrightarrow x = \frac{\pi}{4} \in \left[\frac{\pi}{6}; \frac{\pi}{3} \right]$$

$$\Rightarrow S = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left| \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right| dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left| \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right| dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left| \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right| dx$$

$$= \left| \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right) dx \right| + \left| \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right) dx \right|$$

$$= \left| (tgx + cotgx) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} \right| + \left| (tgx + cotgx) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} \right|.$$

$$\text{Vậy } S = \frac{8\sqrt{3} - 12}{3} \text{ (đvdt).}$$

$$22) \text{ Tọa độ giao điểm } \begin{cases} y = x^2 \\ y = 4x^2 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$\text{Ta có: } \begin{cases} y = x^2 \\ y = 4x^2 \end{cases} \Leftrightarrow \begin{cases} x = \sqrt{y} \\ x = \frac{1}{2}\sqrt{y} \end{cases}$$

$$\Rightarrow S = \left| \int_0^4 \left(\sqrt{y} - \frac{1}{2} \sqrt{y} \right) dy \right| = \left| \frac{\sqrt{y^3}}{3} \right|_0^4.$$

Vậy $S = \frac{8}{3}$ (đvdt).

$$23) S = \int_{-2}^2 |x(x+1)(x-2)| dx$$

$$= \left| \int_{-2}^{-1} (x^3 - x^2 - 2x) dx \right| + \left| \int_{-1}^0 (x^3 - x^2 - 2x) dx \right| + \left| \int_0^2 (x^3 - x^2 - 2x) dx \right|$$

$$= \left| \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right) \right|_{-2}^{-1} + \left| \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right) \right|_{-1}^0 + \left| \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right) \right|_0^2.$$

Vậy $S = \frac{37}{6}$ (đvdt).

$$24) S = \int_{-1}^2 |xe^x| dx = \int_0^2 xe^x dx - \int_{-1}^0 xe^x dx = (x-1)e^x \Big|_0^2 - (x-1)e^x \Big|_{-1}^0.$$

Vậy $S = \frac{e^3 + 2e - 2}{e}$ (đvdt).

$$25) \begin{cases} y^2 = 4x \\ x - y + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{4}y^2 \\ x = y - 1 \end{cases} \Rightarrow \frac{1}{4}y^2 = y - 1 \Leftrightarrow y = 2$$

$$\Rightarrow S = \int_0^2 \left| \frac{1}{4}y^2 - (y-1) \right| dy = \frac{1}{4} \int_0^2 (y^2 - 4y + 4) dy = \frac{1}{4} \left(\frac{y^3}{3} - 2y^2 + 4y \right) \Big|_0^2.$$

Vậy $S = \frac{2}{3}$ (đvdt).

$$26) \begin{cases} x - y^3 + 1 = 0 \\ x + y - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} x = y^3 - 1 \\ x = 1 - y \end{cases} \Rightarrow y^3 - 1 = 1 - y \Leftrightarrow y^3 + y - 2 = 0 \Leftrightarrow y = 1$$

$$\Rightarrow S = \left| \int_0^1 (y^3 + y - 2) dy \right| = \left| \left(\frac{1}{4}y^4 + \frac{1}{2}y^2 - 2y \right) \right|_0^1.$$

Vậy $S = \frac{5}{4}$.

HT 16. Tính thể tích do hình phẳng giới hạn bởi các đường

1) $y = 3x, y = x, x = 0, x = 1$ quay quanh Ox

2) $y = \frac{x^2}{2}, y = 2, y = 4, x = 0$ quay quanh Oy

3) $y^2 = (x-1)^3, x = 2$ và $y = 0$ quay quanh Ox

4) $y^2 = 4 - x, x = 0$ quay quanh Oy

5) (C) : $x^2 + (y-4)^2 = 4$ quay quanh Oy

6) ellipse (E) : $\frac{x^2}{16} + \frac{y^2}{9} = 1$ quay quanh Ox

7) ellipse (E) : $\frac{x^2}{16} + \frac{y^2}{9} = 1$ quay quanh Oy

8) $y = x^2 + 2, y = 4 - x^2$ quay quanh Ox

9) $y = x^2, y = \sqrt{x}$ quay quanh Ox

10) $y = -\sqrt{4-x^2}, x^2 + 3y = 0$ quay quanh Ox

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1)
$$V = \pi \int_0^1 \left| (3x)^2 - x^2 \right| dx = 8\pi \int_0^1 x^2 dx = \frac{8\pi x^3}{3} \Big|_0^1.$$

Vậy $V = \frac{8\pi}{3}$ (đvtt).

2) Ta có $y = \frac{x^2}{2} \Leftrightarrow x^2 = 2y \Rightarrow V = \pi \int_2^4 x^2 dy = \pi \int_2^4 2y dy = \pi y^2 \Big|_2^4.$

Vậy $V = 12\pi$ (đvtt).

3) Ta có $(x-1)^3 = 0 \Leftrightarrow x = 1 \Rightarrow V = \pi \int_1^2 y^2 dx = \pi \int_1^2 (x-1)^3 dx = \pi \frac{(x-1)^4}{4} \Big|_1^2.$

Vậy $V = \frac{\pi}{4}$ (đvtt).

4) Ta có $\begin{cases} y^2 = 4 - x \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} x = 4 - y^2 \\ x = 0 \end{cases} \Rightarrow y = \pm 2$

$$\Rightarrow V = \pi \int_{-2}^2 (4 - y^2)^2 dy = 2\pi \left(16y - \frac{8y^3}{3} + \frac{y^5}{5} \right) \Big|_0^2.$$

Vậy $V = \frac{512\pi}{15}$ (đvtt).

5) Tung độ giao điểm (C) : $x^2 + (y-4)^2 = 4$ và Oy:

$$(y-4)^2 = 4 \Leftrightarrow \begin{cases} y-4 = 2 \\ y-4 = -2 \end{cases} \Leftrightarrow \begin{cases} y = 6 \\ y = 2 \end{cases}$$

$$\Rightarrow V = \pi \int_2^6 x^2 dy = \pi \int_2^6 [4 - (y-4)^2] dy = \pi \left(-\frac{y^3}{3} + 4y^2 - 12y \right) \Big|_2^6.$$

Cách khác:

Hình khối tròn xoay là hình cầu bán kính $R = 2$ nên $V = \frac{4\pi 2^3}{3}$. Vậy $V = \frac{32\pi}{3}$ (đvtt).

6) Hoành độ giao điểm (E): $\frac{x^2}{16} + \frac{y^2}{9} = 1$ và Ox là $x = \pm 4$.

$$\text{Ta có: } \frac{x^2}{16} + \frac{y^2}{9} = 1 \Leftrightarrow y^2 = \frac{9}{16}(16 - x^2)$$

$$\Rightarrow V = \pi \int_{-4}^4 y^2 dx = \frac{9\pi}{16} \int_{-4}^4 (16 - x^2) dx = \frac{9\pi}{8} \left(16x - \frac{x^3}{3} \right) \Big|_0^4.$$

Vậy $V = 48\pi$ (đvtt).

7) Tung độ giao điểm (E): $\frac{x^2}{16} + \frac{y^2}{9} = 1$ và Oy là $y = \pm 3$.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \Leftrightarrow x^2 = \frac{16}{9}(9 - y^2)$$

$$\Rightarrow V = \pi \int_{-4}^4 x^2 dy = \frac{16\pi}{9} \int_{-3}^3 (9 - y^2) dy = \frac{32\pi}{9} \left(9y - \frac{y^3}{3} \right) \Big|_0^3.$$

Vậy $V = 64\pi$ (đvtt).

8) Hoành độ giao điểm $x^2 + 2 = 4 - x^2 \Leftrightarrow x = \pm 1$

$$\Rightarrow V = \pi \int_{-1}^1 \left| (x^2 + 2)^2 - (4 - x^2)^2 \right| dx = 24\pi \int_0^1 |x^2 - 1| dx = 24\pi \left| \left(\frac{x^3}{3} - x \right) \right|_0^1.$$

Vậy $V = 16\pi$ (đvtt).

9) Hoành độ giao điểm $x^2 = \sqrt{x} \Leftrightarrow x^4 = x \Leftrightarrow x = 0 \vee x = 1$

$$\Rightarrow V = \pi \int_0^1 |x^4 - x| dx = \pi \left| \int_0^1 (x^4 - x) dx \right| = \pi \left| \left(\frac{x^5}{5} - \frac{x^2}{2} \right) \right|_0^1.$$

Vậy $V = \frac{3\pi}{10}$ (đvtt).

10) Hoành độ giao điểm $-\sqrt{4-x^2} = -\frac{x^2}{3} \Leftrightarrow x^2 = 3 \Leftrightarrow x = \pm\sqrt{3}$

$$\Rightarrow V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} \left((4-x^2) - \frac{x^4}{9} \right) dx = \frac{2\pi}{9} \int_0^{\sqrt{3}} (36 - 3x^3 - x^4) dx = \frac{2\pi}{9} \left(36x - 3x^3 - \frac{x^5}{5} \right) \Big|_0^{\sqrt{3}}$$

Vậy $V = \frac{28\pi\sqrt{3}}{5}$ (đvtt).

Xin chân thành cảm ơn quý thầy cô và các bạn học sinh đã đọc tài liệu này!

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