Nơi khởi đầu ước mơ

TUYỂN TẬP LƯỢNG GIÁC

(ĐÁP ÁN CHI TIẾT)

BIÊN SOẠN: LƯU HUY THƯỞNG

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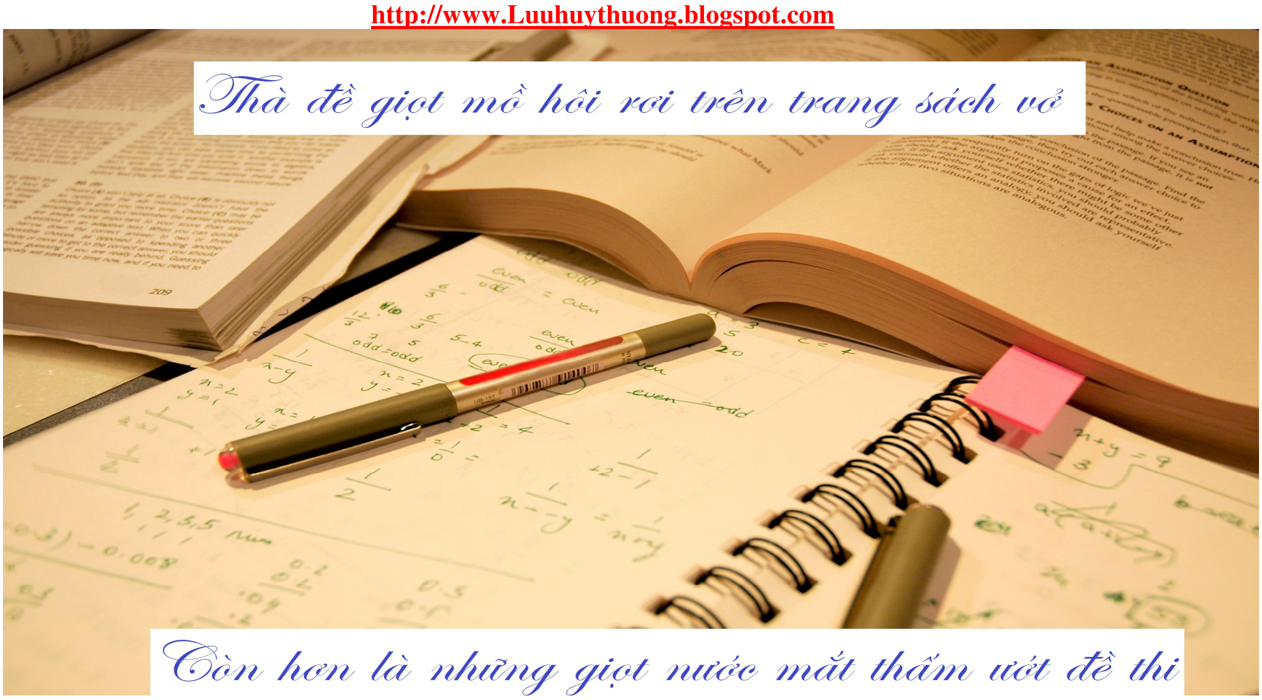

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TUYỂN TẬP LƯỢNG GIÁC

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HT 1.Giải các phương trình:

1) $2\cos^2 x + \sqrt{3}\cos x = 0$ 2) $\sin^2 x + \sin 2x + 2\cos^2 x = 2$ 3) $3\sin^2 x + \sin 2x + \cos^2 x = 3$

4) $2\sin^2 x - \sin x - 1 = 0$ 5) $\cos 2x + 3\sin x - 2 = 0$ 6) $2\cos 2x - 3\cos x + 1 = 0$

Bài giải

1) $2\cos^2 x + \sqrt{3}\cos x = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ \cos x = -\dfrac{\sqrt{3}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k\pi \\ x = \pm\dfrac{5\pi}{6} + k2\pi \end{array}\right., k \in \mathbb{Z}$$

2) $\sin^2 x + \sin 2x + 2\cos^2 x = 2$

$$\Leftrightarrow \sin x(2\cos x - \sin x) = 0 \Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \tan x = 2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \arctan 2 + k\pi \end{array}\right.$$

3) $3\sin^2 x + \sin 2x + \cos^2 x = 3$

$$\Leftrightarrow 2\sin x\cos x - 2\cos^2 x = 0 \Leftrightarrow 2\cos x(\sin x - \cos x) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ \tan x = 1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k2\pi \\ x = \dfrac{\pi}{4} + k\pi \end{array}\right.$$

4) $$2\sin^2 x - \sin x - 1 = 0 \Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ \sin x = -\dfrac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k2\pi \\ x = -\dfrac{\pi}{6} + k2\pi, k \in \mathbb{Z} \\ x = \dfrac{7\pi}{6} + k2\pi \end{array}\right.$$

5) $\cos 2x + 3\sin x - 2 = 0$

$$\Leftrightarrow 1 - 2\sin^2 x + 3\sin x - 2 = 0 \Leftrightarrow 2\sin^2 x - 3\sin x + 1 = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ \sin x = \dfrac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k2\pi \\ x = \dfrac{\pi}{6} + k2\pi \;, k \in \mathbb{Z} \\ x = \dfrac{5\pi}{6} + k2\pi \end{array}\right.$$

6) $2\cos 2x - 3\cos x + 1 = 0 \Leftrightarrow 4\cos^2 x - 3\cos x - 1 = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 1 \\ \cos x = -\dfrac{1}{4} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k2\pi \\ x = \pm\arccos(-\dfrac{1}{4}) + k2\pi \end{array}\right., k \in \mathbb{Z}$$

HT 2. Giải các phương trình sau:

1) $\sqrt{3}\sin 3x - \cos 3x = 2$

2) $\sin 5x + \cos 5x = -\sqrt{2}$

3) $\sqrt{3}\sin x + \cos x = \sqrt{2}$

4) $\sqrt{3}\sin x - \cos x = \sqrt{2}$

Bài giải

1) $\sqrt{3}\sin 3x - \cos 3x = 2$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\sin 3x - \frac{1}{2}\cos 3x = 1 \Leftrightarrow \sin\ (3x - \frac{\pi}{6}) = 1 \Leftrightarrow 3x - \frac{\pi}{6} = \frac{\pi}{2} + k2\pi \Leftrightarrow x = \frac{2\pi}{9} + \frac{k2\pi}{3}$$

2) $\sin 5x + \cos 5x = -\sqrt{2}$

$$\Leftrightarrow \frac{1}{\sqrt{2}}\sin 5x + \frac{1}{\sqrt{2}}\cos 5x = -1 \Leftrightarrow \sin\ (5x + \frac{\pi}{4}) = -1 \Leftrightarrow 5x + \frac{\pi}{4} = -\frac{\pi}{2} + k2\pi \Leftrightarrow x = -\frac{3\pi}{20} + \frac{k2\pi}{5}$$

3) $\sqrt{3}\sin x + \cos x = \sqrt{2} \Leftrightarrow \dfrac{\sqrt{3}}{2}\sin x + \dfrac{1}{2}\cos x = \dfrac{\sqrt{2}}{2}$

$$\Leftrightarrow \sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2} \Leftrightarrow \sin(x + \frac{\pi}{6}) = \sin\frac{\pi}{4}$$

$$\Leftrightarrow \left[\begin{array}{l} x + \dfrac{\pi}{6} = \dfrac{\pi}{4} + k2\pi \\ x + \dfrac{\pi}{6} = \dfrac{3\pi}{4} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{12} + k2\pi \\ x = \dfrac{7\pi}{12} + k2\pi \end{array}\right., k \in \mathbb{Z}$$

4) $\sqrt{3}\sin x - \cos x = \sqrt{2} \Leftrightarrow \dfrac{\sqrt{3}}{2}\sin x - \dfrac{1}{2}\cos x = \dfrac{\sqrt{2}}{2}$

$$\Leftrightarrow \sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2} \Leftrightarrow \sin(x - \frac{\pi}{6}) = \sin\frac{\pi}{4}$$

$$\Leftrightarrow \left[\begin{array}{l} x - \dfrac{\pi}{6} = \dfrac{\pi}{4} + k2 \\ x - \dfrac{\pi}{6} = \dfrac{3\pi}{4} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{5\pi}{12} + k2\pi \\ x = \dfrac{11\pi}{12} + k2\pi \end{array}\right., k \in \mathbb{Z}$$

HT 3. Giải phương trình:

1) $3\sin 3x - \sqrt{3}\cos 9x = 1 + 4\sin^3 3x$

2) $\tan x - \sin 2x - \cos 2x + 2(2\cos x - \dfrac{1}{\cos x}) = 0$

3) $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$

4) $9\sin x+6\cos x-3\sin 2x+\cos 2x=8$

5) $\sin 2x+2\cos 2x=1+\sin x-4\cos x$

6) $2\sin 2x-\cos 2x=7\sin x+2\cos x-4$

7) $\sin 2x-\cos 2x=3\sin x+\cos x-2$

8) $(\sin 2x+\sqrt{3}\cos 2x)^2-5=\cos(2x-\frac{\pi}{6})$

9) $2\cos^3 x+\cos 2x+\sin x=0$

10) $1+\cot 2x=\frac{1-\cos 2x}{\sin^2 2x}$

11) $4(\sin^4 x+\cos^4 x)+\sqrt{3}\sin 4x=2$

12) $1+\sin^3 2x+\cos^3 2x=\frac{1}{2}\sin 4x$

13) $\tan x-3\cot x=4(\sin x+\sqrt{3}\cos x)$

14) $\sin^3 x+\cos^3 x=\sin x-\cos x$

15) $\cos^4 x+\sin^4(x+\frac{\pi}{4})=\frac{1}{4}$

16) $4\sin^3 x\cos 3x+4\cos^3 x\sin 3x+3\sqrt{3}\cos 4x=3$

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Bài giải

1) $3\sin 3x-\sqrt{3}\cos 9x=1+4\sin^3 3x \Leftrightarrow (3\sin 3x-4\sin^3 3x)-\sqrt{3}\cos 9x=1$

$$\Leftrightarrow \sin 9x-\sqrt{3}\cos 9x=1 \Leftrightarrow \sin(9x-\frac{\pi}{3})=\sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{18}+k\frac{2\pi}{9} \\ x=\frac{7\pi}{54}+k\frac{2\pi}{9}\end{array}\right.$$

2) $\tan x-\sin 2x-\cos 2x+2(2\cos x-\frac{1}{\cos x})=0$ (1)

Điều kiện: $\cos x\neq 0 \Leftrightarrow x\neq\frac{\pi}{2}+k\pi$

$$(1)\Leftrightarrow \frac{\sin x}{\cos x}-\sin 2x-\cos 2x+4\cos x-\frac{2}{\cos x}=0$$

$$\Leftrightarrow \sin x-2\sin x\cos^2 x-\cos 2x\cos x+2(2\cos^2 x-1)=0$$

$$\Leftrightarrow \sin x(1-2\cos^2 x)-\cos 2x\cos x+2\cos 2x=0$$

$$\Leftrightarrow -\sin x\cos 2x-\cos 2x\cos x+2\cos 2x=0$$

$$\Leftrightarrow \cos 2x(\sin x+\cos x-2)=0 \Leftrightarrow \left[\begin{array}{l} \cos 2x=0 \\ \sin x+\cos x=2 (vn)\end{array}\right. \Leftrightarrow x=\frac{\pi}{4}+k\frac{\pi}{2}$$

3) $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x}$ (*)

Điều kiện: $\sin 2x \neq 0 \Leftrightarrow x \neq k\dfrac{\pi}{2}$

$(*) \Leftrightarrow 8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x \;\; \Leftrightarrow 4(1-\cos 2x)\cos x = \sqrt{3}\sin x + \cos x$

$\Leftrightarrow -4\cos 2x\cos x = \sqrt{3}\sin x - 3\cos x \;\; \Leftrightarrow -2(\cos 3x + \cos x) = \sqrt{3}\sin x - 3\cos x$

$\Leftrightarrow \cos 3x = \dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x \;\; \Leftrightarrow \cos 3x = \cos(x + \dfrac{\pi}{3}) \;\; \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{6} + k\pi \\ x = -\dfrac{\pi}{12} + k\dfrac{\pi}{2} \end{array}\right.$

C2 $(*) \Leftrightarrow 8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x \;\; \Leftrightarrow 8(1-\cos^2 x)\cos x = \sqrt{3}\sin x + \cos x$

$\Leftrightarrow 8\cos x - 8\cos^3 x = \sqrt{3}\sin x - 3\cos x \;\; \Leftrightarrow 6\cos x - 8\cos^3 x = \sqrt{3}\sin x - \cos x$

$\Leftrightarrow 4\cos^3 x - 3\cos x = \dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x \;\; \Leftrightarrow \cos 3x = \cos(x + \dfrac{\pi}{3})$

$\Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{6} + k\pi \\ x = -\dfrac{\pi}{12} + k\dfrac{\pi}{2} \end{array}\right.$

4) $9\sin x + 6\cos x - 3\sin 2x + \cos 2x = 8$

$\Leftrightarrow 6\sin x\cos x - 6\cos x + 2\sin^2 x - 9\sin x + 7 = 0$

$\Leftrightarrow 6\cos x(\sin x - 1) + (\sin x - 1)(2\sin x - 7) = 0$

$\Leftrightarrow (\sin x - 1)(6\cos x + 2\sin x - 7) = 0$

$\Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ 6\cos x + 2\sin x = 7 \end{array}\right. \;\; \Leftrightarrow x = \dfrac{\pi}{2} + k2\pi$

5) $\sin 2x + 2\cos 2x = 1 + \sin x - 4\cos x$

$\Leftrightarrow 2\sin x\cos x + 2(2\cos^2 x - 1) - 1 - \sin x + 4\cos x = 0$

$\Leftrightarrow \sin x(2\cos x - 1) + 4\cos^2 x + 4\cos x - 3 = 0$

$\Leftrightarrow \sin x(2\cos x - 1) + (2\cos x - 1)(2\cos x + 3) = 0$

$\Leftrightarrow (2\cos x - 1)(2\sin x + 2\cos x + 3) = 0$

$\Leftrightarrow \left[\begin{array}{l} \cos x = \dfrac{1}{2} \\ 2\sin x + 2\cos x = -3, (vn) \end{array}\right. \;\; \Leftrightarrow x = \pm\dfrac{\pi}{3} + k2\pi$

6) $2\sin 2x - \cos 2x = 7\sin x + 2\cos x - 4$

$$\Leftrightarrow 4\sin x\cos x-(1-2\sin^2 x)-7\sin x-2\cos x+4=0$$

$$\Leftrightarrow 2\cos x(2\sin x-1)+(2\sin^2 x-7\sin x+3)=0$$

$$\Leftrightarrow 2\cos x(2\sin x-1)+(2\sin x-1)(\sin x-3)=0$$

$$\Leftrightarrow (2\sin x-1)(2\cos x+\sin x-3)=0$$

$$\Leftrightarrow \left[\begin{array}{l} 2\sin x-1=0 \\ 2\cos x+\sin x=3,(vn) \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{6}+k2\pi \\ x=\frac{5\pi}{6}+k2\pi \end{array}\right.$$

7) $\sin 2x-\cos 2x=3\sin x+\cos x-2$

$$\Leftrightarrow 2\sin x\cos x-(1-2\sin^2 x)-3\sin x-\cos x+2=0$$

$$\Leftrightarrow (2\sin x\cos x-\cos x)+(2\sin^2 x-3\sin x+1)=0$$

$$\Leftrightarrow \cos x(2\sin x-1)+(2\sin x-1)(\sin x-1)=0$$

$$\Leftrightarrow (2\sin x-1)(\cos x+\sin x-1)=0 \Leftrightarrow \left[\begin{array}{l} 2\sin x=1 \\ \cos x+\sin x=1 \end{array}\right.$$

$$+2\sin x=1 \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{6}+k2\pi \\ x=\frac{5\pi}{6}+k2\pi \end{array}\right.$$

$$+\cos x+\sin x=1 \Leftrightarrow \cos(x-\frac{\pi}{4})=\frac{\sqrt{2}}{2} \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right.$$

8) $(\sin 2x+\sqrt{3}\cos 2x)^2-5=\cos(2x-\frac{\pi}{6})$

Ta có: $\sin 2x+\sqrt{3}\cos 2x=2(\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x)=2\cos(2x-\frac{\pi}{6})$

Đặt: $t=\sin 2x+\sqrt{3}\cos 2x, -2\le t\le 2$

Phương trình trở thành: $t^2-5=\frac{t}{2} \Leftrightarrow 2t^2-t-10=0 \Leftrightarrow \left[\begin{array}{l} t=-2 \\ t=\frac{5}{2} \end{array}\right.$

$+t=\frac{5}{2}$: loại

$+t=-2 : 2\cos(2x-\frac{\pi}{6})=-2 \Leftrightarrow x=\frac{7\pi}{12}+k\pi$

9) $2\cos^3 x + \cos 2x + \sin x = 0 \Leftrightarrow 2\cos^3 x + 2\cos^2 x - 1 + \sin x = 0$

$\Leftrightarrow 2\cos^2 x(\cos x + 1) - (1 - \sin x) = 0 \Leftrightarrow 2(1 - \sin^2 x)(\cos x + 1) - (1 - \sin x) = 0$

$\Leftrightarrow 2(1 - \sin x)(1 + \sin x)(\cos x + 1) - (1 - \sin x) = 0$

$\Leftrightarrow (1 - \sin x)[2(1 + \sin x)(\cos x + 1) - 1] = 0$

$\Leftrightarrow (1 - \sin x)[1 + 2\sin x\cos x + 2(\sin x + \cos x)] = 0$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ 1 + 2\sin x\cos x + 2(\sin x + \cos x) = 0 \end{array}\right.$$

$+\sin x = 1 \Leftrightarrow x = \dfrac{\pi}{2} + k2\pi$

$+1 + 2\sin x\cos x + 2(\sin x + \cos x) = 0 \Leftrightarrow (\sin x + \cos x)^2 + 2(\sin x + \cos x) = 0$

$\Leftrightarrow (\sin x + \cos x)(\sin x + \cos x + 2) = 0 \Leftrightarrow \sin x + \cos x = 0$

$\Leftrightarrow \tan x = -1 \Leftrightarrow x = -\dfrac{\pi}{4} + k\pi$

10) $1 + \cot 2x = \dfrac{1 - \cos 2x}{\sin^2 2x}$ (*) Điều kiện: $\sin 2x \neq 0 \Leftrightarrow x \neq k\dfrac{\pi}{2}$

$(*) \Leftrightarrow 1 + \cot 2x = \dfrac{1 - \cos 2x}{1 - \cos^2 2x} \Leftrightarrow 1 + \cot 2x = \dfrac{1}{1 + \cos 2x} \Leftrightarrow 1 + \dfrac{\cos 2x}{\sin 2x} = \dfrac{1}{1 + \cos 2x}$

$\Leftrightarrow \sin 2x(1 + \cos 2x) + \cos 2x(1 + \cos 2x) = \sin 2x$

$\Leftrightarrow \sin 2x\cos 2x + \cos 2x(1 + \cos 2x) = 0 \Leftrightarrow \cos 2x(\sin 2x + \cos 2x + 1) = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos 2x = 0 \\ \sin 2x + \cos 2x = -1 \end{array}\right.$$

$+\cos 2x = 0 \Leftrightarrow x = \dfrac{\pi}{4} + k\dfrac{\pi}{2}$

$+\sin 2x + \cos 2x = -1 \Leftrightarrow \sin(2x + \dfrac{\pi}{4}) = \sin(-\dfrac{\pi}{4}) \Leftrightarrow \left[\begin{array}{l} x = -\dfrac{\pi}{4} + k\pi \\ x = \dfrac{\pi}{2} + k\pi \end{array}\right.$

Vậy,phương trình có nghiệm: $x = \dfrac{\pi}{4} + k\dfrac{\pi}{2}$

11) $4(\sin^4 x + \cos^4 x) + \sqrt{3}\sin 4x = 2$

$\Leftrightarrow 4[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x] + \sqrt{3}\sin 4x = 2$

$\Leftrightarrow 4(1-\frac{1}{2}\sin^2 2x)+\sqrt{3}\sin 4x=2 \ \Leftrightarrow \cos 4x+\sqrt{3}\sin 4x=-2$

$\Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+k\frac{\pi}{2} \\ x=-\frac{\pi}{12}+k\frac{\pi}{2} \end{array}\right.$

12) $1+\sin^3 2x+\cos^3 2x=\frac{1}{2}\sin 4x$

$\Leftrightarrow 2-\sin 4x+2(\sin 2x+\cos 2x)(1-\sin 2x\cos 2x)=0$

$\Leftrightarrow (2-\sin 4x)+(\sin 2x+\cos 2x)(2-\sin 4x)=0$

$\Leftrightarrow (2-\sin 4x)(\sin 2x+\cos 2x+1)=0 \ \Leftrightarrow \sin 2x+\cos 2x=-1$

$\Leftrightarrow \sin(2x+\frac{\pi}{4})=-\frac{\sqrt{2}}{2} \ \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+k\pi \\ x=\frac{\pi}{2}+k\pi \end{array}\right.$

13) $\tan x-3\cot x=4(\sin x+\sqrt{3}\cos x)$ (*) Điều kiện: $\sin 2x\neq 0 \Leftrightarrow x\neq k\frac{\pi}{2}$

$(*)\Leftrightarrow \frac{\sin x}{\cos x}-3\frac{\cos x}{\sin x}=4(\sin x+\sqrt{3}\cos x)$

$\Leftrightarrow \sin^2 x-3\cos^2 x-4\sin x\cos x(\sin x+\sqrt{3}\cos x)=0$

$\Leftrightarrow (\sin x-\sqrt{3}\cos x)(\sin x+\sqrt{3}\cos x)-4\sin x\cos x(\sin x+\sqrt{3}\cos x)=0$

$\Leftrightarrow (\sin x+\sqrt{3}\cos x)(\sin x-\sqrt{3}\cos x-4\sin x\cos x)=0$

$\Leftrightarrow \left[\begin{array}{l} \sin x+\sqrt{3}\cos x=0 \\ \sin x-\sqrt{3}\cos x-4\sin x\cos x=0 \end{array}\right.$

$+\sin x+\sqrt{3}\cos x=0 \Leftrightarrow \tan x=-\sqrt{3} \Leftrightarrow x=-\frac{\pi}{3}+k\pi$

$+\sin x-\sqrt{3}\cos x-4\sin x\cos x=0 \Leftrightarrow 2\sin 2x=\sin x-\sqrt{3}\cos x$

$\Leftrightarrow \sin 2x=\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x \ \Leftrightarrow \sin 2x=\sin(x-\frac{\pi}{3}) \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{3}+k2\pi \\ x=\frac{4\pi}{9}+k\frac{2\pi}{3} \end{array}\right.$

Vậy,phương trình có nghiệm là: $x=-\frac{\pi}{3}+k\pi;\ x=\frac{4\pi}{9}+k\frac{2\pi}{3}$

14) $\sin^3 x+\cos^3 x=\sin x-\cos x \Leftrightarrow \sin x(\sin^2 x-1)+\cos^3 x+\cos x=0$

$\Leftrightarrow -\sin x\cos^2 x+\cos^3 x+\cos x=0 \Leftrightarrow \cos x(-\sin x\cos x+\cos^2 x+1)=0$

$\Leftrightarrow \left[\begin{array}{c}\cos x=0\\ -\sin x\cos x+\cos^2 x=-1\end{array}\right.$

$+\cos x=0\Leftrightarrow x=\frac{\pi}{2}+k\pi$

$+-\sin x\cos x+\cos^2 x=-1\Leftrightarrow -\frac{1}{2}\sin 2x+\frac{1+\cos 2x}{2}=-1\Leftrightarrow \sin 2x-\cos 2x=3,(vn)$

Vậy,phương trình có nghiệm là: $x=\frac{\pi}{2}+k\pi,k\in\mathbb{Z}$

15) $\cos^4 x+\sin^4(x+\frac{\pi}{4})=\frac{1}{4} \Leftrightarrow \frac{1}{4}(1+\cos 2x)^2+\frac{1}{4}[1-\cos(2x+\frac{\pi}{2})]^2=\frac{1}{4}$

$\Leftrightarrow (1+\cos 2x)^2+(1+\sin 2x)^2=1 \Leftrightarrow \sin 2x+\cos 2x=-1$

$\Leftrightarrow \cos(2x-\frac{\pi}{4})=\cos\frac{3\pi}{4} \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=-\frac{\pi}{4}+k\pi\end{array}\right.$

16) $4\sin^3 x\cos 3x+4\cos^3 x\sin 3x+3\sqrt{3}\cos 4x=3$

$\Leftrightarrow 4\sin^3 x(4\cos^3 x-3\cos x)+4\cos^3 x(3\sin x-4\sin^3 x)+3\sqrt{3}\cos 4x=3$

$\Leftrightarrow -12\sin^3 x\cos x+12\cos^3 x\sin x+3\sqrt{3}\cos 4x=3$

$\Leftrightarrow 4\sin x\cos x(\cos^2 x-\sin^2 x)+\sqrt{3}\cos 4x=1$

$\Leftrightarrow 2\sin 2x\cos 2x+\sqrt{3}\cos 4x=1\Leftrightarrow \sin 4x+\sqrt{3}\cos 4x=1$

$\Leftrightarrow \frac{1}{2}\sin 4x+\frac{\sqrt{3}}{2}\cos 4x=\frac{1}{2}\Leftrightarrow \sin(4x+\frac{\pi}{3})=\sin\frac{\pi}{6} \quad \Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{24}+k\frac{\pi}{2}\\ x=\frac{\pi}{8}+k\frac{\pi}{2}\end{array}\right.,k\in\mathbb{Z}$

HT 4.Giải phương trình:

1) $\cos^4 x+\sin^4 x+\cos(x-\frac{\pi}{4})\sin(3x-\frac{\pi}{4})-\frac{3}{2}=0$

2) $5\sin x-2=3(1-\sin x)\tan^2 x$

3) $2\sin 3x-\frac{1}{\sin x}=2\cos 3x+\frac{1}{\cos x}$

4) $\frac{\cos x(2\sin x+3\sqrt{2})-2\cos^2 x-1}{1+\sin 2x}=1$

5) $\cos x\cos\frac{x}{2}\cos\frac{3x}{2}-\sin x\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2}$

6) $4\cos^3 x+3\sqrt{2}\sin 2x=8\cos x$

7) $\cos(2x+\frac{\pi}{4})+\cos(2x-\frac{\pi}{4})+4\sin x=2+\sqrt{2}(1-\sin x)$ **8)** $3\cot^2 x+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

9) $\frac{4\sin^2 2x+6\sin^2 x-9-3\cos 2x}{\cos x}=0$ **10)** $\cos x+\cos 3x+2\cos 5x=0$

11) $\sin^8 x+\cos^8 x=\frac{17}{16}\cos^2 2x$ **12)** $\sin\frac{5x}{2}=5\cos^3 x\sin\frac{x}{2}$

13) $\sin 2x(\cot x+\tan 2x)=4\cos^2 x$ **14)** $\tan^3(x-\frac{\pi}{4})=\tan x-1$

15) $\frac{\sin^4 2x+\cos^4 2x}{\tan(\frac{\pi}{4}-x)\tan(\frac{\pi}{4}+x)}=\cos^4 4x$ **16)** $48-\frac{1}{\cos^4 x}-\frac{2}{\sin^2 x}(1+\cot 2x\cot x)=0$

17) $\sin^8 x+\cos^8 x=2(\sin^{10} x+\cos^{10} x)+\frac{5}{4}\cos 2x$

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Bài giải

1) $\cos^4 x+\sin^4 x+\cos(x-\frac{\pi}{4})\sin(3x-\frac{\pi}{4})-\frac{3}{2}=0$

$\Leftrightarrow (\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x+\frac{1}{2}[\sin(4x-\frac{\pi}{2})+\sin 2x]-\frac{3}{2}=0$

$\Leftrightarrow 1-\frac{1}{2}\sin^2 2x+\frac{1}{2}(-\cos 4x+\sin 2x)-\frac{3}{2}=0$

$\Leftrightarrow -\frac{1}{2}\sin^2 2x-\frac{1}{2}(1-2\sin^2 2x)+\frac{1}{2}\sin 2x-\frac{1}{2}=0$

$\Leftrightarrow \sin^2 2x+\sin 2x-2=0 \Leftrightarrow \sin 2x=1 \Leftrightarrow x=\frac{\pi}{4}+k\pi$

2) $5\sin x-2=3(1-\sin x)\tan^2 x$ (1)

Điều kiện: $\cos x\neq 0\Leftrightarrow x\neq\frac{\pi}{2}+k\pi$

$(1)\Leftrightarrow 5\sin x-2=3(1-\sin x)\frac{\sin^2 x}{\cos^2 x}\Leftrightarrow 5\sin x-2=3(1-\sin x)\frac{\sin^2 x}{1-\sin^2 x}$

$\Leftrightarrow 5\sin x-2=\frac{3\sin^2 x}{1+\sin x}\Leftrightarrow 2\sin^2 x+3\sin x-2=0\Leftrightarrow \sin x=\frac{1}{2}$

$$\Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{6} + k2\pi \\ x = \dfrac{5\pi}{6} + k2\pi \end{array}\right.$$

3) $2\sin 3x - \dfrac{1}{\sin x} = 2\cos 3x + \dfrac{1}{\cos x}$ (*)

Điều kiện: $\sin 2x \neq 0 \Leftrightarrow x \neq k\dfrac{\pi}{2}$

$$(*) \Leftrightarrow 2(\sin 3x - \cos 3x) = \frac{1}{\sin x} + \frac{1}{\cos x}$$

$$\Leftrightarrow 2[3(\sin x + \cos x) - 4(\sin^3 x + \cos^3 x] = \frac{1}{\sin x} + \frac{1}{\cos x}$$

$$\Leftrightarrow 2(\sin x + \cos x)[3 - 4(\sin^2 x - \sin x\cos x + \cos^2 x)] = \frac{\sin x + \cos x}{\sin x\cos x}$$

$$\Leftrightarrow 2(\sin x + \cos x)(-1 + 4\sin x\cos x) - \frac{\sin x + \cos x}{\sin x\cos x} = 0$$

$$\Leftrightarrow (\sin x + \cos x)(-2 + 8\sin x\cos x - \frac{1}{\sin x\cos x}) = 0$$

$$\Leftrightarrow (\sin x + \cos x)(4\sin 2x - \frac{2}{\sin 2x} - 2) = 0$$

$$\Leftrightarrow (\sin x + \cos x)(4\sin^2 2x - 2\sin 2x - 2) = 0$$

$$\Leftrightarrow \left[\begin{array}{c} \sin x + \cos x = 0 \\ 4\sin^2 2x - 2\sin 2x - 2 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{c} \tan x = -1 \\ \sin 2x = 1 \\ \sin 2x = -1/2 \end{array}\right. \Leftrightarrow \left[\begin{array}{c} x = \pm\dfrac{\pi}{4} + k\pi \\ x = -\dfrac{\pi}{12} + k\pi \\ x = \dfrac{7\pi}{12} + k\pi \end{array}\right.$$

4) $\dfrac{\cos x(2\sin x + 3\sqrt{2}) - 2\cos^2 x - 1}{1 + \sin 2x} = 1$ (*)

Điều kiện: $\sin 2x \neq -1 \Leftrightarrow x \neq -\dfrac{\pi}{4} + k\pi$

$$(*) \Leftrightarrow 2\sin x\cos x + 3\sqrt{2}\cos x - 2\cos^2 x - 1 = 1 + \sin 2x$$

$$\Leftrightarrow 2\cos^2 x - 3\sqrt{2}\cos x + 2 = 0 \Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow x = \pm\frac{\pi}{4} + k\pi$$

Đối chiếu điều kiện phương trình có nghiệm: $x = \dfrac{\pi}{4} + k\pi, k \in \mathbb{Z}$

5) $\cos x\cos\frac{x}{2}\cos\frac{3x}{2}-\sin x\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2}$

$$\Leftrightarrow \frac{1}{2}\cos x(\cos 2x+\cos x)+\frac{1}{2}\sin x(\cos 2x-\cos x)=\frac{1}{2}$$

$$\Leftrightarrow \cos x\cos 2x+\cos^2 x+\sin x\cos 2x-\sin x\cos x=1$$

$$\Leftrightarrow \cos 2x(\sin x+\cos x)+1-\sin^2 x-\sin x\cos x-1=0$$

$$\Leftrightarrow \cos 2x(\sin x+\cos x)-\sin x(\sin x+\cos x)=0$$

$$\Leftrightarrow (\sin x+\cos x)(\cos 2x-\sin x)=0$$

$$\Leftrightarrow (\sin x+\cos x)(-2\sin^2 x-\sin x+1)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ 2\sin^2 x+\sin x-1=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\tan x=-1\\ \sin x=-1\\ \sin x=1/2\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=-\frac{\pi}{2}+k2\pi\\ x=\frac{\pi}{6}+k2\pi\vee x=\frac{5\pi}{6}+k2\pi\end{array}\right.$$

6) $4\cos^3 x+3\sqrt{2}\sin 2x=8\cos x \Leftrightarrow 4\cos^3 x+6\sqrt{2}\sin x\cos x-8\cos x=0$

$$\Leftrightarrow 2\cos x(2\cos^2 x+3\sqrt{2}\sin x-4)=0 \Leftrightarrow 2\cos x(2\sin^2 x-3\sqrt{2}\sin x+2)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \sin x=\frac{\sqrt{2}}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\ x=\frac{\pi}{4}+k2\pi\\ x=\frac{3\pi}{4}+k2\pi\end{array}\right.$$

7) $\cos(2x+\frac{\pi}{4})+\cos(2x-\frac{\pi}{4})+4\sin x=2+\sqrt{2}(1-\sin x)$

$$\Leftrightarrow 2\cos 2x\cos\frac{\pi}{4}+4\sin x-2-\sqrt{2}+\sqrt{2}\sin x=0$$

$$\Leftrightarrow \sqrt{2}(1-2\sin^2 x)+4\sin x-2-\sqrt{2}+\sqrt{2}\sin x=0$$

$$\Leftrightarrow 2\sqrt{2}\sin^2 x-(4+\sqrt{2})\sin x+2=0$$

$$\Leftrightarrow \sin x=\frac{1}{2} \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{6}+k2\pi\\ x=\frac{5\pi}{6}+k2\pi\end{array}\right.$$

8) $3\cot^2 x + 2\sqrt{2}\sin^2 x = (2+3\sqrt{2})\cos x$ (1)

Điều kiện: $\sin x \neq 0 \Leftrightarrow x \neq k\pi$

$$(1) \Leftrightarrow 3\frac{\cos^2 x}{\sin^4 x} + 2\sqrt{2} = (2+3\sqrt{2})\frac{\cos x}{\sin^2 x}$$

Đặt: $t = \dfrac{\cos x}{\sin^2 x}$ phương trình trở thành: $3t^2 - (2+3\sqrt{2})t + 2\sqrt{2} = 0 \Leftrightarrow \left[\begin{array}{l} t = \sqrt{2} \\ t = \dfrac{2}{3}\end{array}\right.$

$+t = \dfrac{2}{3} : \dfrac{\cos x}{\sin^2 x} = \dfrac{2}{3} \Leftrightarrow 3\cos x = 2(1-\cos^2 x) \Leftrightarrow 2\cos^2 x + 3\cos x - 2 = 0$

$$\Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} + k2\pi$$

$+t = \sqrt{2} : \dfrac{\cos x}{\sin^2 x} = \sqrt{2} \Leftrightarrow \cos x = \sqrt{2}(1-\cos^2 x) \Leftrightarrow \sqrt{2}\cos^2 x + \cos x - \sqrt{2} = 0$

$$\Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow x = \pm\frac{\pi}{4} + k2\pi$$

Vậy,phương trình có nghiệm: $x = \pm\dfrac{\pi}{3} + k2\pi, x = \pm\dfrac{\pi}{4} + k2\pi$

9) $\dfrac{4\sin^2 2x + 6\sin^2 x - 9 - 3\cos 2x}{\cos x} = 0$ (*)

Điều kiện: $\cos x \neq 0 \Leftrightarrow x \neq \dfrac{\pi}{2} + k\pi$

$(*) \Leftrightarrow 4(1-\cos^2 2x) + 3(1-\cos 2x) - 9 - 3\cos x = 0 \Leftrightarrow 4\cos^2 2x + 6\cos x + 2 = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos 2x = -1 \\ \cos 2x = -\dfrac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k\pi \\ x = \pm\dfrac{\pi}{3} + k\pi\end{array}\right.$$

Vậy,phương trình có nghiệm: $x = \pm\dfrac{\pi}{3} + k\pi$

10) $\cos x + \cos 3x + 2\cos 5x = 0 \Leftrightarrow (\cos 5x + \cos x) + (\cos 5x + \cos 3x) = 0$

$$\Leftrightarrow 2\cos 3x\cos 2x + 2\cos 4x\cos x = 0$$

$$\Leftrightarrow (4\cos^3 x - 3\cos x)\cos 2x + (2\cos^2 2x - 1)\cos x = 0$$

$$\Leftrightarrow \cos x[(4\cos^2 x - 3)\cos 2x + 2\cos^2 2x - 1] = 0$$

$$\Leftrightarrow \cos x\{[2(1+\cos 2x) - 3]\cos 2x + 2\cos^2 2x - 1\} = 0$$

$\Leftrightarrow \cos x(4\cos^2 2x - \cos 2x - 1) = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ \cos x = \dfrac{1-\sqrt{17}}{8} \\ \cos x = \dfrac{1+\sqrt{17}}{8} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k\pi \\ x = \pm \arccos\dfrac{1-\sqrt{17}}{8} + k2\pi \\ x = \pm \arccos\dfrac{1+\sqrt{17}}{8} + k2\pi \end{array}\right.$$

11) $\sin^8 x + \cos^8 x = \dfrac{17}{16}\cos^2 2x$ (*)

$\sin^8 x + \cos^8 x = (\sin^4 x + \cos^4 x)^2 - 2\sin^4 x\cos^4 x$

$= [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x)]^2 - \dfrac{1}{8}\sin^4 2x$

$= (1 - \dfrac{1}{2}\sin^2 2x)^2 - \dfrac{1}{8}\sin^4 2x = 1 - \sin^2 2x + \dfrac{1}{8}\sin^4 2x$

$(*) \Leftrightarrow 16(1 - \sin^2 2x + \dfrac{1}{8}\sin^4 2x) = 17(1 - \sin^2 2x) \Leftrightarrow 2\sin^4 2x + \sin^2 2x - 1 = 0$

$\Leftrightarrow \sin^2 2x = \dfrac{1}{2} \Leftrightarrow 1 - 2\sin^2 2x = 0 \Leftrightarrow \cos 4x = 0 \Leftrightarrow x = \dfrac{\pi}{8} + k\dfrac{\pi}{4}$

12) $\sin\dfrac{5x}{2} = 5\cos^3 x\sin\dfrac{x}{2}$ (*)

Ta thấy: $\cos\dfrac{x}{2} = 0 \Leftrightarrow x = \pi + k2\pi \Leftrightarrow \cos x = -1$

Thay vào phương trình (*) ta được:

$\sin(\dfrac{5\pi}{2} + 5k\pi) = -\sin(\dfrac{\pi}{2} + k\pi)$ không thỏa mãn với mọi k

Do đó $\cos\dfrac{x}{2}$ không là nghiệm của phương trình nên:

$(*) \Leftrightarrow \sin\dfrac{5x}{2}\cos\dfrac{x}{2} = 5\cos^3 x\sin\dfrac{x}{2}\cos\dfrac{x}{2} \Leftrightarrow \dfrac{1}{2}(\sin 3x + \sin 2x) = \dfrac{5}{2}\cos^3 x\sin x$

$\Leftrightarrow 3\sin x - 4\sin^3 x + 2\sin x\cos x - 5\cos^3 x\sin x = 0$

$\Leftrightarrow \sin x(3 - 4\sin^2 x + 2\cos x - 5\cos^3 x) = 0$

$\Leftrightarrow \sin x(5\cos^3 x - 4\cos^2 x - 2\cos x + 1) = 0$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \cos x = 1 \\ \cos x = \dfrac{-1+\sqrt{21}}{10} \\ \cos x = \dfrac{-1-\sqrt{21}}{10} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = k2\pi \\ x = \pm\arccos\dfrac{-1+\sqrt{21}}{10} + k2\pi \\ x = \pm\arccos\dfrac{-1-\sqrt{21}}{10} + k2\pi \end{array}\right.$$

Vậy,phương trình có nghiệm: $x = k2\pi$, $x = \pm\arccos\dfrac{-1+\sqrt{21}}{10} + k2\pi$

$$x = \pm\arccos\frac{-1-\sqrt{21}}{10} + k2\pi$$

13) $\sin 2x(\cot x + \tan 2x) = 4\cos^2 x$ (1)

Điều kiện: $\begin{cases} \sin x \neq 0 \\ \cos 2x \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq k\pi \\ x \neq \dfrac{\pi}{4} + k\dfrac{\pi}{2} \end{cases}$

Ta có: $\cot x + \tan 2x = \dfrac{\cos x}{\sin x} + \dfrac{\sin 2x}{\cos 2x} = \dfrac{\cos 2x\cos x + \sin 2x\sin x}{\sin x\cos 2x} = \dfrac{\cos x}{\sin x\cos 2x}$

$$(1) \Leftrightarrow 2\sin x\cos x\frac{\cos x}{\sin x\cos 2x} = 4\cos^2 x$$

$$\Leftrightarrow \frac{\cos^2 x}{\cos 2x} = 2\cos^2 x \Leftrightarrow \cos^2 x(1 - 2\cos 2x) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ \cos 2x = 1/2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{2} + k\pi \\ x = \pm\dfrac{\pi}{6} + k\pi \end{array}\right.$$

Vậy,phương trình có nghiệm: $x = \dfrac{\pi}{2} + k\pi$, $x = \pm\dfrac{\pi}{6} + k\pi$

Vậy,phương trình có nghiệm: $x = k\dfrac{5\pi}{2}$, $x = \pm\dfrac{5}{4}\arccos\dfrac{1-\sqrt{21}}{4} + k\dfrac{5\pi}{2}$

14) $\tan^3(x - \dfrac{\pi}{4}) = \tan x - 1$ (1)

Điều kiện: $\begin{cases} \cos x \neq 0 \\ \cos(x - \dfrac{\pi}{4}) \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq \dfrac{\pi}{2} + k\pi \\ x \neq \dfrac{3\pi}{4} + k\pi \end{cases}$

$$(1) \Leftrightarrow \frac{(\tan x - 1)^3}{(1 + \tan x)^3} = \tan x - 1 \Leftrightarrow (\tan x - 1)^3 = (\tan x - 1)(1 + \tan x)^3$$

$\Leftrightarrow (\tan x-1)[(1+\tan x)^3-(\tan x-1)^2]=0 \qquad \Leftrightarrow (\tan x-1)(\tan^3 x+2\tan^2 x+5\tan x)=0$

$\Leftrightarrow \tan x(\tan x-1)(\tan^2 x+2\tan x+5)=0$

$$\Leftrightarrow \left[\begin{array}{l}\tan x=0\\ \tan x=1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=k\pi\\ x=\dfrac{\pi}{4}+k\pi\end{array}\right.$$

C2: Đặt: $t=x-\dfrac{\pi}{4}$

15) $\dfrac{\sin^4 2x+\cos^4 2x}{\tan(\frac{\pi}{4}-x)\tan(\frac{\pi}{4}+x)}=\cos^4 4x \qquad (1)$

Điều kiện: $\begin{cases}\sin(\frac{\pi}{4}-x)\cos(\frac{\pi}{4}-x)\neq 0\\ \sin(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}+x)\neq 0\end{cases} \Leftrightarrow \begin{cases}\sin(\frac{\pi}{4}-2x)\neq 0\\ \sin(\frac{\pi}{4}+2x)\neq 0\end{cases} \Leftrightarrow \cos 2x\neq 0$

$\tan(\frac{\pi}{4}-x)\tan(\frac{\pi}{4}+x)=\dfrac{1-\tan x}{1+\tan x}.\dfrac{1+\tan x}{1-\tan x}=1$

$(1)\Leftrightarrow \sin^4 2x+\cos^4 2x=\cos^4 4x \Leftrightarrow 1-2\sin^2 2x\cos^2 2x=\cos^4 4x$

$\Leftrightarrow 1-\dfrac{1}{2}\sin^2 4x=\cos^4 4x \Leftrightarrow 1-\dfrac{1}{2}(1-\cos^2 4x)=\cos^4 4x$

$\Leftrightarrow 2\cos^4 4x-\cos^2 4x-1=0 \Leftrightarrow \cos^2 4x=1$

$\Leftrightarrow 1-\cos^2 4x=0 \Leftrightarrow \sin 4x=0 \Leftrightarrow x=k\dfrac{\pi}{4}$

Vậy,phương trình có nghiệm: $x=k\dfrac{\pi}{2}$

16) $48-\dfrac{1}{\cos^4 x}-\dfrac{2}{\sin^2 x}(1+\cot 2x\cot x)=0 \qquad (*)$

Điều kiện: $\sin 2x\neq 0 \Leftrightarrow x\neq k\dfrac{\pi}{2}$

Ta có: $1+\cot 2x\cot x=1+\dfrac{\cos 2x\cos x}{\sin 2x\sin x}=\dfrac{\cos 2x\sin x+\sin 2x\sin x}{\sin 2x\cos x}$

$$=\dfrac{\cos x}{2\sin^2 x\cos x}=\dfrac{1}{2\sin^2 x}$$

$(*)\Leftrightarrow 48-\dfrac{1}{\cos^4 x}-\dfrac{1}{\sin^4 x}=0 \Leftrightarrow 48=\dfrac{1}{\cos^4 x}+\dfrac{1}{\sin^4 x}$

$\Leftrightarrow 48\sin^4 x\cos^4 x = \sin^4 x + \cos^4 x \Leftrightarrow 3\sin^4 2x = 1 - \frac{1}{2}\sin^2 2x$

$\Leftrightarrow 6\sin^4 2x + \sin^2 2x - 2 = 0 \Leftrightarrow \sin^2 2x = \frac{1}{2} \Leftrightarrow 1 - 2\sin^2 2x = 0$

$\Leftrightarrow \cos 4x = 0 \Leftrightarrow x = \frac{\pi}{8} + k\frac{\pi}{4}$

Vậy,phương trình có nghiệm: $x = \frac{\pi}{8} + k\frac{\pi}{4}$

17) $\sin^8 x + \cos^8 x = 2(\sin^{10} x + \cos^{10} x) + \frac{5}{4}\cos 2x$

$\Leftrightarrow \sin^8 x(1 - 2\sin^2 x) - \cos^8 x(2\cos^2 x - 1) = \frac{5}{4}\cos 2x$

$\Leftrightarrow \sin^8 x\cos 2x - \cos^8 x\cos 2x = \frac{5}{4}\cos 2x$

$\Leftrightarrow 4\cos 2x(\cos^8 x - \sin^8 x) + 5\cos 2x = 0$

$\Leftrightarrow 4\cos 2x(\cos^4 x - \sin^4 x)(\cos^4 x + \sin^4 x) + 5\cos 2x = 0$

$\Leftrightarrow 4\cos 2x(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)(\cos^4 x + \sin^4 x) + 5\cos 2x = 0$

$\Leftrightarrow 4\cos 2x(\cos^2 x - \sin^2 x)(1 - \frac{1}{2}\sin^2 2x) + 5\cos 2x = 0$

$\Leftrightarrow 4\cos^2 2x(1 - \frac{1}{2}\sin^2 2x) + 5\cos 2x = 0 \Leftrightarrow 4\cos 2x(4\cos 2x - 2\cos 2x\sin^2 2x + 5) = 0$

$\Leftrightarrow 4\cos 2x[4\cos 2x - 2\cos 2x(1 - \cos^2 2x) + 5] = 0$

$\Leftrightarrow 4\cos 2x(2\cos^3 2x + 2\cos 2x + 5) = 0 \Leftrightarrow \cos 2x = 0 \Leftrightarrow x = \frac{\pi}{4} + k\frac{\pi}{2}$

HT 5.Giải các phương trình sau:

1) $\frac{\sin^4 x + \cos^4 x}{\sin 2x} = \frac{1}{2}\left(\tan x + \cot x\right)$

2) $1 + \sin\frac{x}{2}\sin x - \cos\frac{x}{2}\sin^2 x = 2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

3) $\sin(2x + \frac{17\pi}{2}) + 16 = 2\sqrt{3}.\sin x\cos x + 20\sin^2(\frac{x}{2} + \frac{\pi}{12})$

4) $\sin x + \sin^2 x + \sin^3 x + \sin^4 x = \cos x + \cos^2 x + \cos^3 x + \cos^4 x$

5) $2\sqrt{2}\cos\left(\frac{5\pi}{12} - x\right)\sin x = 1$

6) $\frac{\sin 2x}{\sin x + \cos x} + \frac{1}{\sqrt{2}.\tan x} = 2\cos x$

7) $\cos^2 x + \sin x \sin 4x - \sin^2 4x = \frac{1}{4}$

8) $2\cos 4x - (\sqrt{3} - 2)\cos 2x = \sin 2x + \sqrt{3}$

9) $1 + \sin x - \cos x - \sin 2x + \cos 2x = 0$

10) $\sin x \cos 2x + \cos^2 x\left(\tan^2 x - 1\right) + 2\sin^3 x = 0$

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Bài giải

1) $\dfrac{\sin^4 x + \cos^4 x}{\sin 2x} = \dfrac{1}{2}\left(\tan x + \cot x\right)$ (1)

Điều kiện: $\sin 2x \neq 0$

$(1) \Leftrightarrow \dfrac{1 - \frac{1}{2}\sin^2 2x}{\sin 2x} = \dfrac{1}{2}\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right) \Leftrightarrow \dfrac{1 - \frac{1}{2}\sin^2 2x}{\sin 2x} = \dfrac{1}{\sin 2x} \quad \Leftrightarrow 1 - \dfrac{1}{2}\sin^2 2x = 1 \quad \Leftrightarrow \sin 2x = 0$

Vậy phương trình đã cho vô nghiệm.

2) $1 + \sin\dfrac{x}{2}\sin x - \cos\dfrac{x}{2}\sin^2 x = 2\cos^2\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right) \qquad (1)$

$(1) \Leftrightarrow 1 + \sin\dfrac{x}{2}\sin x - \cos\dfrac{x}{2}\sin^2 x = 1 + \cos\left(\dfrac{\pi}{2} - x\right) = 1 + \sin x$

$\Leftrightarrow \sin x\left(\sin\dfrac{x}{2} - \cos\dfrac{x}{2}\sin x - 1\right) = 0 \Leftrightarrow \sin x\left(\sin\dfrac{x}{2} - \cos\dfrac{x}{2}.2\sin\dfrac{x}{2}\cos\dfrac{x}{2} - 1\right) = 0$

$\Leftrightarrow \sin x\left(\sin\dfrac{x}{2} - 1\right)\left(2\sin^2\dfrac{x}{2} + 2\sin\dfrac{x}{2} + 1\right) = 0 \Leftrightarrow \sin x = 0, \sin\dfrac{x}{2} = 1, 2\sin^2\dfrac{x}{2} + 2\sin\dfrac{x}{2} + 1 = 0$

$\Leftrightarrow x = k\pi, \dfrac{x}{2} = \dfrac{\pi}{2} + k2\pi \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \pi + k4\pi \end{array}\right. \Leftrightarrow x = k\pi$

3) $\sin(2x + \dfrac{17\pi}{2}) + 16 = 2\sqrt{3}.\sin x\cos x + 20\sin^2(\dfrac{x}{2} + \dfrac{\pi}{12})$

Biến đổi phương trình đó cho tương đương với

$\cos 2x - \sqrt{3}\sin 2x + 10\cos(x + \dfrac{\pi}{6}) + 6 = 0 \Leftrightarrow \cos(2x + \dfrac{\pi}{3}) + 5\cos(x + \dfrac{\pi}{6}) + 3 = 0$

$\Leftrightarrow 2\cos^2(x + \dfrac{\pi}{6}) + 5\cos(x + \dfrac{\pi}{6}) + 2 = 0$ .Giải được $\cos(x + \dfrac{\pi}{6}) = -\dfrac{1}{2}$ và $\cos(x + \dfrac{\pi}{6}) = -2$ (loại)

*Giải $\cos(x + \dfrac{\pi}{6}) = -\dfrac{1}{2}$ được nghiệm $x = \dfrac{\pi}{2} + k2\pi$ và $x = -\dfrac{5\pi}{6} + k2\pi$

4) $\sin x + \sin^2 x + \sin^3 x + \sin^4 x = \cos x + \cos^2 x + \cos^3 x + \cos^4 x$

$\sin x+\sin^2 x+\sin^3 x+\sin^4 x=\cos x+\cos^2 x+\cos^3 x+\cos^4 x$

$\sin x+\sin^2 x+\sin^3 x+\sin^4 x=\cos x+\cos^2 x+\cos^3 x+\cos^4 x$

$\Leftrightarrow (\sin x-cosx).\left[2+2(\sin x+cosx)+\sin x.cosx\right]=0 \Leftrightarrow \left[\begin{array}{l}\sin x-cosx=0\\ 2+2(\sin x+cosx)+\sin x.cosx=0\end{array}\right.$

+ Với $\sin x-cosx=0 \Leftrightarrow x=\frac{\pi}{4}+k\pi \ \ (k\in Z)$

+ Với $2+2(\sin x+cosx)+\sin x.cosx=0$, đặt t = $\sin x+cosx$ (t $\in\left[-\sqrt{2};\sqrt{2}\right]$)

được pt : $t^2+4t=3=0 \Leftrightarrow \left[\begin{array}{l}t=-1\\ t=-3(loai)\end{array}\right.$ t = -1 $\Rightarrow \left[\begin{array}{l}x=\pi+m2\pi\\ x=-\frac{\pi}{2}+m2\pi\end{array}\right.(m\in Z)$

Vậy : $x=\frac{\pi}{4}+k\pi, x=\pi+m2\pi, x=-\frac{\pi}{2}+m2\pi(m\in Z, k\in Z)$

5) $2\sqrt{2}\cos\left(\frac{5\pi}{12}-x\right)\sin x=1$

$2\sqrt{2}cos\left(\frac{5\pi}{12}-x\right)\sin x=1 \Leftrightarrow \sqrt{2}\left[\sin\left(2x-\frac{5\pi}{12}\right)+\sin\frac{5\pi}{12}\right]=1$

$\Leftrightarrow \sin\left(2x-\frac{5\pi}{12}\right)+\sin\frac{5\pi}{12}=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4} \Leftrightarrow \sin\left(2x-\frac{5\pi}{12}\right)=\sin\frac{\pi}{4}-\sin\frac{5\pi}{12}=$
$=2\cos\frac{\pi}{3}\sin\left(-\frac{\pi}{12}\right)=\sin\left(-\frac{\pi}{12}\right)$

$\Leftrightarrow \sin\left(2x-\frac{5\pi}{12}\right)=\sin\left(-\frac{\pi}{12}\right) \Leftrightarrow \left[\begin{array}{l}2x-\frac{5\pi}{12}=-\frac{\pi}{12}+k2\pi\\ 2x-\frac{5\pi}{12}=\frac{13\pi}{12}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{6}+k\pi\\ x=\frac{3\pi}{4}+k\pi\end{array}\right.\left(k\in\mathbb{Z}\right)$

6) $\frac{\sin 2x}{\sin x+\cos x}+\frac{1}{\sqrt{2}.\tan x}=2cosx$

Điều kiện: $\sin x\neq 0,\ \cos x\neq 0, \sin x+\cos x\neq 0.$

Pt đã cho trở thành $\frac{\cos x}{\sqrt{2}\sin x}+\frac{2\sin x\cos x}{\sin x+\cos x}-2\cos x=0$

$\Leftrightarrow \frac{\cos x}{\sqrt{2}\sin x}-\frac{2\cos^2 x}{\sin x+\cos x}=0 \Leftrightarrow \cos x\left(\sin(x+\frac{\pi}{4})-\sin 2x\right)=0$

+) $\cos x=0 \Leftrightarrow x=\frac{\pi}{2}+k\pi,\ \ k\in$ Z.

+) $\sin 2x = \sin(x + \frac{\pi}{4}) \Leftrightarrow \left[\begin{array}{l} 2x = x + \frac{\pi}{4} + m2\pi \\ 2x = \pi - x - \frac{\pi}{4} + n2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + m2\pi \\ x = \frac{\pi}{4} + \frac{n2\pi}{3} \end{array}\right. \quad m, n \in Z \Leftrightarrow x = \frac{\pi}{4} + \frac{t2\pi}{3}, \ t \in Z.$

Đối chiếu điều kiện ta có nghiệm của pt là : $x = \frac{\pi}{2} + k\pi$; $x = \frac{\pi}{4} + \frac{t2\pi}{3}, \ k, t \in Z.$

7) $\cos^2 x + \sin x \sin 4x - \sin^2 4x = \frac{1}{4}$

pt đã cho tương đương với pt:

$\frac{1}{2}(1 + \cos 2x) + \frac{1}{2}(\cos 3x - \cos 5x) - \frac{1}{2}(1 - \cos 8x) = \frac{1}{4}$

$\Leftrightarrow \cos 3x \cos 5x + \frac{1}{2}\cos 3x - \frac{1}{2}\left(\cos 5x + \frac{1}{2}\right) = 0$

$\Leftrightarrow \left(\cos 5x + \frac{1}{2}\right)\left(\cos 3x - \frac{1}{2}\right) = 0 \Leftrightarrow \left[\begin{array}{l} \cos 5x + \frac{1}{2} = 0 \\ \cos 3x - \frac{1}{2} = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \pm\frac{2\pi}{15} + k\frac{2\pi}{5} \\ x = \pm\frac{\pi}{9} + k\frac{2\pi}{3} \end{array}\right.$

8) $2\cos 4x - (\sqrt{3} - 2)\cos 2x = \sin 2x + \sqrt{3}$

$\Leftrightarrow 2(\cos 4x + \cos 2x) = (\cos 2x + 1) + \sin 2x$

$\Leftrightarrow 4\cos 3x.\cos x = 2\sqrt{3}\cos^2 x + 2\sin x \cos x \Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ 2\cos 3x = \sqrt{3}\cos x + \sin x \end{array}\right.$

+ $\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi$

+ $2\cos 3x = \sqrt{3}\cos x + \sin x \Leftrightarrow \cos 3x = \cos\left(x - \frac{\pi}{6}\right) \Leftrightarrow \left[\begin{array}{l} 3x = x - \frac{\pi}{6} + k2\pi \\ 3x = \frac{\pi}{6} - x + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{12} + k\pi \\ x = \frac{\pi}{24} + \frac{k\pi}{2} \end{array}\right.$

9) $1 + \sin x - \cos x - \sin 2x + \cos 2x = 0$

$\Leftrightarrow (1 - \sin 2x) + (\sin x - \cos x) + (\cos^2 x - \sin^2 x) = 0$

$\Leftrightarrow (\sin x - \cos x)\left[(\sin x - \cos x) + 1 - (\sin x + \cos x)\right] = 0$

$\Leftrightarrow ((\sin x - \cos x)(1 - 2\cos x) = 0$

$\Leftrightarrow \left[\begin{array}{l} \tan x = 1 \\ \cos x = \frac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + k.\pi \\ x = \pm\frac{\pi}{3} + l.\pi \end{array}\right. \left(k, l \in \mathbb{Z}\right)$ (k,l ∈ Z).

10) $\sin x\cos 2x+\cos^2 x\left(\tan^2 x-1\right)+2\sin^3 x=0$

Điều kiện $\cos x\neq 0$

$\sin x\cos 2x+\cos^2 x\left(\tan^2 x-1\right)+2\sin^3 x=0$

$\Leftrightarrow \sin x\left(1-2\sin^2 x\right)+2\sin^2 x-1+2\sin^3 x=0$

$\Leftrightarrow 2\sin^2 x+\sin x-1=0\Leftrightarrow \left[\begin{array}{l}\sin x=-1\\ \sin x=\dfrac{1}{2}\end{array}\right.\Leftrightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{2}+k2\pi\\ x=\dfrac{\pi}{6}+k2\pi\\ x=\dfrac{5\pi}{6}+k2\pi\end{array}\right.$.

Kết hợp điều kiện, phương trình có nghiệm $S=\left\{\dfrac{\pi}{6}+k2\pi;\dfrac{5\pi}{6}+k2\pi\right\}$

HT 6.Giải các phương trình sau:

1) $\sqrt{2}.\cos 2x=\dfrac{1}{\sin x}+\dfrac{1}{\cos x}\ (1)$

2) $2\cos 3x.\cos x+\sqrt{3}(1+\sin 2x)=2\sqrt{3}\cos^2(2x+\dfrac{\pi}{4})$

3) $\cos x+\cos 3x=1+\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)$

4) $\dfrac{1}{\tan x+\cot 2x}=\dfrac{\sqrt{2}(\sin x-\cos x)}{\cot x-1}$

5) $\dfrac{4\sqrt{3}\sin x\cos^2 x-2\cos\dfrac{5x}{2}\cos\dfrac{x}{2}+\sqrt{3}\sin 2x+3\cos x+2}{2\sin x-\sqrt{3}}=0\ (1)$

6) $2\sin 2x+\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)+5\sin x-3\cos x=3$

7) $(\tan x+1)\sin^2 x+\cos 2x+2=3(\cos x+\sin x)\sin x.$

8) $\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)=3\sin x+\cos x+2$

9) $\dfrac{\left(1+\sin x\right)\left(5-2\sin x\right)}{\left(2\sin x+3\right)\cos x}=\sqrt{3}$

10) $\tan 2x-\tan x=\dfrac{1}{6}(\sin 4x+\sin 2x)(1)$

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Bài giải

1) $\sqrt{2}.\cos 2x=\dfrac{1}{\sin x}+\dfrac{1}{\cos x}\ (1)$ Điều kiện: $x\neq k\dfrac{\pi}{2}$

$(1)\Leftrightarrow \sqrt{2}.\cos 2x-\dfrac{\cos x+\sin x}{\sin x.\cos x}=0$

$\Leftrightarrow \frac{\sqrt{2}}{2}(\cos x - \sin x)(\cos x + \sin x)\sin 2x - (\cos x + \sin x) = 0$

$\Leftrightarrow (\cos x + \sin x)\left[(\cos x - \sin x)\sin 2x - \sqrt{2}\right] = 0$

$\Leftrightarrow \left[\begin{array}{l}\cos x + \sin x = 0\\(\cos x - \sin x)\sin 2x - \sqrt{2} = 0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 0\\(\cos x - \sin x)\left(1 - (\cos x - \sin x)^2\right) - \sqrt{2} = 0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\sin\left(x + \frac{\pi}{4}\right) = 0\\(\cos x - \sin x)^3 - (\cos x - \sin x) + \sqrt{2} = 0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = \frac{-\pi}{4} + k\pi\\x = \frac{3\pi}{4} + k2\pi\end{array}\right.$

ĐS: $x = \frac{-\pi}{4} + k\pi$, $k \in Z$

2) $2\cos 3x.\cos x + \sqrt{3}(1 + \sin 2x) = 2\sqrt{3}\cos^2(2x + \frac{\pi}{4})$

$PT \Leftrightarrow \cos 4x + \cos 2x + \sqrt{3}(1 + \sin 2x) = \sqrt{3}\left(1 + \cos(4x + \frac{\pi}{2})\right) \Leftrightarrow \cos 4x + \sqrt{3}\sin 4x + \cos 2x + \sqrt{3}\sin 2x = 0$

$\Leftrightarrow \sin(4x + \frac{\pi}{6}) + \sin(2x + \frac{\pi}{6}) = 0 \Leftrightarrow 2\sin(3x + \frac{\pi}{6}).\cos x = 0 \Leftrightarrow \left[\begin{array}{l}x = -\frac{\pi}{18} + k\frac{\pi}{3}\\x = \frac{\pi}{2} + k\pi\end{array}\right.$

Vậy PT có hai nghiệm $x = \frac{\pi}{2} + k\pi$ và $x = -\frac{\pi}{18} + k\frac{\pi}{3}$.

3) $\cos x + \cos 3x = 1 + \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$

$\Leftrightarrow 2\cos 2x\cos x = 1 + \sin 2x + \cos 2x \Leftrightarrow \cos 2x(2\cos x - 1) = 1 + 2\sin x\cos x$

$\Leftrightarrow (\cos^2 x - \sin^2 x)(2\cos x - 1) = (\cos x + \sin x)^2 \Leftrightarrow \left[\begin{array}{ll}\cos x + \sin x = 0 & (1)\\(\cos x - \sin x)(2\cos x - 1) = \cos x + \sin x & (2)\end{array}\right.$

$(1) \Leftrightarrow \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 0 \Leftrightarrow x + \frac{\pi}{4} = k\pi \Leftrightarrow x = -\frac{\pi}{4} + k\pi$

$(2) \Leftrightarrow 2\cos x(\cos x - \sin x - 1) = 0 \Leftrightarrow \left[\begin{array}{l}\cos x = 0\\\sqrt{2}\cos\left(x + \frac{\pi}{4}\right) = 1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = \frac{\pi}{2} + k\pi\\x + \frac{\pi}{4} = \pm\frac{\pi}{4} + k2\pi\end{array}\right.$

Vậy pt có nghiệm là $x = -\frac{\pi}{4} + k\pi$, $x = \frac{\pi}{2} + k\pi$, $x = k2\pi$

4) $\frac{1}{\tan x+\cot 2x}=\frac{\sqrt{2}(\sin x-\cos x)}{\cot x-1}$

Điều kiện : sinx.cosx $\begin{cases} \text{s inx}.\cos x\neq 0\\ \cot x\neq 1\end{cases}$

Phương trình đã cho tương đương với phương trình:

$$\frac{1}{\frac{\sin x}{\cos x}+\frac{\cos 2x}{\sin 2x}}=\frac{\sqrt{2}\left(\sin x-\cos x\right)}{\frac{\cos x-\sin x}{\sin x}}$$

$$\Leftrightarrow \frac{\cos x.\sin 2x}{\cos x}=\frac{\sqrt{2}(\sin x-\cos x)\sin x}{\cos x-\sin x}$$

$$\Leftrightarrow \cos x=-\frac{\sqrt{2}}{2}\Leftrightarrow \left[\begin{array}{l} x=-\frac{3\pi}{4}+k2\pi\\ x=\frac{3\pi}{4}+k2\pi\end{array}\right.(k\in Z)$$

Đối chiếu điều kiện ta được nghiệm của phương trình là: $x=\frac{3\pi}{4}+k2\pi,(k\in Z)$

5) $\frac{4\sqrt{3}\sin x\cos^2 x-2\cos\frac{5x}{2}\cos\frac{x}{2}+\sqrt{3}\sin 2x+3\cos x+2}{2\sin x-\sqrt{3}}=0\ (1)$

Điều kiện : $\sin x\neq\frac{\sqrt{3}}{2}$

$$2\sqrt{3}\sin 2x\cos x-\cos 3x-\cos 2x+\sqrt{3}\sin 2x+3\cos x+2=0$$
$$\Leftrightarrow \sqrt{3}\sin 2x\left(2\cos x+1\right)-\left(\cos 3x-\cos x\right)-\left(\cos 2x-1\right)+2\cos x+1=0$$

$$\Leftrightarrow \sqrt{3}\sin 2x\left(2\cos x+1\right)+4\cos x.\sin^2 x+2\sin^2 x+2\cos x+1=0$$

$$\Leftrightarrow \sqrt{3}\sin 2x\left(2\cos x+1\right)+2\sin^2 x\left(2\cos x+1\right)+\left(2\cos x+1\right)=0$$

$$\Leftrightarrow \left(2\cos x+1\right)\left(\sqrt{3}\sin 2x+2\sin^2 x+1\right)=0\Leftrightarrow \left(2\cos x+1\right)\left(\sqrt{3}\sin 2x-\cos 2x+2\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l} 2\cos x+1=0\\ \sqrt{3}\sin 2x-\cos 2x+2=0\end{array}\right.\Leftrightarrow \left[\begin{array}{l} \cos x=\frac{-1}{2}\\ \cos\left(2x+\frac{\pi}{3}\right)=\frac{1}{2}\end{array}\right.\Leftrightarrow \left[\begin{array}{l} x=\pm\frac{2\pi}{3}+2k\pi\\ x=k\pi;x=\frac{-\pi}{3}+k\pi\end{array}\right.\left(k\in \mathrm{Z}\right)$$

Đối chiếu điều kiện ta được nghiệm của phương trình là: $x=k\pi;x=\frac{-2\pi}{3}+k2\pi;x=\frac{-\pi}{3}+k2\pi(k\in Z)$

6) $2\sin 2x+\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)+5\sin x-3\cos x=3$ (1)

$(1) \Leftrightarrow 2\sin 2x + \sin 2x + cos2x + 5\sin x - 3\cos x = 3$

$\Leftrightarrow 6\sin x\cos x - 3\cos x - (2\sin^2 x - 5\sin x + 2) = 0$

$\Leftrightarrow 3\cos x(2\sin x - 1) - (2\sin x - 1)(\sin x - 2) = 0$

$\Leftrightarrow (2\sin x - 1)(3\cos x - \sin x + 2) = 0$

$\Leftrightarrow \sin x = \dfrac{1}{2}, \sin x - 3\cos x = 2$

+ $\sin x = \dfrac{1}{2} \Leftrightarrow x = \dfrac{\pi}{6} + k2\pi, x = \dfrac{5\pi}{6} + k2\pi; k \in \mathbb{Z}$

$\text{s inx} - 3\cos x = 2 \Leftrightarrow \sin(x - \alpha) = \dfrac{2}{\sqrt{10}}, (\cos\alpha = \dfrac{1}{\sqrt{10}}) \Leftrightarrow x = \alpha + \arcsin\dfrac{2}{\sqrt{10}} + k2\pi$

$x = \pi + \alpha - \arcsin\dfrac{2}{\sqrt{10}} + k2\pi, k \in \mathbb{Z}$

Vậy pt có 4 họ nghiệm :

$x = \dfrac{\pi}{6} + k2\pi, x = \dfrac{5\pi}{6} + k2\pi, x = \alpha + \arcsin\dfrac{2}{\sqrt{10}} + k2\pi, \pi + \alpha - \arcsin\dfrac{2}{\sqrt{10}} + k2\pi; k \in \mathbb{Z}$

7) $(\tan x + 1)\sin^2 x + \cos 2x + 2 = 3(\cos x + \sin x)\sin x.$

Điều kiện: $\cos x \neq 0,$ hay $x \neq \dfrac{\pi}{2} + k\pi.$

Khi đó phương trình đã cho tương đương với

$(\tan x + 1)\sin^2 x + 1 - 2\sin^2 x + 2 = 3(\cos x + \sin x)\sin x \Leftrightarrow (\tan x - 1)\sin^2 x + 3 = 3(\cos x - \sin x)\sin x + 6\sin^2 x$

$\Leftrightarrow (\tan x - 1)\sin^2 x + 3\cos 2x = 3(\cos x - \sin x)\sin x$

$\Leftrightarrow (\tan x - 1)\sin^2 x + 3(\cos x - \sin x)\cos x = 0$

$\Leftrightarrow (\sin x - \cos x)(\sin^2 x - 3\cos^2 x) = 0 \Leftrightarrow (\sin x - \cos x)(2\cos 2x + 1) = 0$

$\Leftrightarrow \left[\begin{array}{l} \sin x = \cos x \\ \cos 2x = -\dfrac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{4} + k\pi \\ x = \pm\dfrac{\pi}{3} + k\pi,\ k \in � . \end{array}\right.$

Đối chiếu điều kiện ta có nghiệm $x = \dfrac{\pi}{4} + k\pi,\ x = \pm\dfrac{\pi}{3} + k\pi,\ k \in Z$

8) $\sqrt{2}\sin\left(2x + \dfrac{\pi}{4}\right) = 3\sin x + \cos x + 2$

$\Leftrightarrow \sin 2x + \cos 2x = 3\sin x + \cos x + 2$

$\Leftrightarrow 2\sin x\cos x + 2\cos^2 x - 1 = 3\sin x + \cos x + 2$

$\Leftrightarrow \sin x\left(2\cos x - 3\right) + 2\cos^2 x - \cos x - 3 = 0$

$\Leftrightarrow \sin x\left(2\cos x - 3\right) + \left(\cos x + 1\right)\left(2\cos x - 3\right) = 0 \Leftrightarrow \left(2\cos x - 3\right)\left(\sin x + \cos x + 1\right) = 0$

$\Leftrightarrow \sin x + \cos x + 1 = 0 \Leftrightarrow \sin x + \cos x = -1 \Leftrightarrow \sin\left(x + \dfrac{\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}$

$\Leftrightarrow \left[\begin{array}{l} x + \dfrac{\pi}{4} = -\dfrac{\pi}{4} + k2\pi \\ x + \dfrac{\pi}{4} = \dfrac{5\pi}{4} + k2\pi \end{array}\right.$, ($k \in Z$) $\Leftrightarrow \left[\begin{array}{l} x = -\dfrac{\pi}{2} + k2\pi \\ x = \pi + k2\pi \end{array}\right.$ ($k \in Z$.)

9) $\dfrac{\left(1 + \sin x\right)\left(5 - 2\sin x\right)}{\left(2\sin x + 3\right)\cos x} = \sqrt{3}$

$\cos x \neq 0 \Leftrightarrow x \neq \dfrac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\dfrac{\left(1 + \sin x\right)\left(5 - 2\sin x\right)}{\left(2\sin x + 3\right)\cos x} = \sqrt{3} \Leftrightarrow 5 + 3\sin x - 2\sin^2 x = \sqrt{3}\sin 2x + 3\sqrt{3}\cos x \Leftrightarrow$

$\left(\cos 2x - \sqrt{3}\sin 2x\right) + 3\left(\sin x - \sqrt{3}\cos x\right) + 4 = 0 \Leftrightarrow \cos\left(2x + \dfrac{\pi}{3}\right) - 3\cos\left(x + \dfrac{\pi}{6}\right) + 2 = 0$

$\Leftrightarrow 2\cos^2\left(x + \dfrac{\pi}{6}\right) - 3\cos\left(x + \dfrac{\pi}{6}\right) + 1 = 0 \Leftrightarrow \left[\begin{array}{l} \cos\left(x + \dfrac{\pi}{6}\right) = 1 \\ \cos\left(x + \dfrac{\pi}{6}\right) = \dfrac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\dfrac{\pi}{6} + k2\pi \\ x = \dfrac{\pi}{6} + k2\pi \quad ,k \in \mathbb{Z} \\ x = -\dfrac{\pi}{2} + k2\pi \end{array}\right.$

Đối chiếu điều kiện ta có các nghiệm $x = \pm\dfrac{\pi}{6} + k2\pi, k \in \mathbb{Z}$

10) $\tan 2x - \tan x = \dfrac{1}{6}(\sin 4x + \sin 2x)(1)$

Điều kiện: $\begin{cases} \cos 2x \neq 0 \\ \cos x \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq \dfrac{\pi}{4} + \dfrac{m\pi}{2} \\ x \neq \dfrac{\pi}{2} + m\pi \end{cases} m \in Z$

$(1) \Leftrightarrow 6\sin x = \cos 2x\cos x(\sin 4x + \sin 2x)$

$\Leftrightarrow 6\sin x = \cos x\cos 2x(4\sin x\cos x\cos 2x + 2\sin x\cos x)$

$\Leftrightarrow \sin x(4\cos^2 x\cos^2 2x + 2\cos^2 x\cos 2x - 6) = 0$

$\Leftrightarrow \sin x\left[(2\cos^2 2x(1 + \cos 2x) + \cos 2x(1 + \cos 2x) - 6\right] = 0$

$\Leftrightarrow \sin x(2\cos^3 2x + 3\cos^2 2x + \cos 2x - 6) = 0$

$\Leftrightarrow \sin x(\cos 2x - 1)(2\cos^2 2x + 5\cos 2x + 6) = 0$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ \cos 2x = 1 \\ 2\cos^2 2x + 5\cos 2x + 6 = 0 (VN) \end{array} \right. \Leftrightarrow x = k\pi (tm) k \in Z$$

HT 7.Giải các phương trình sau:

1) $2(\sin x-\cos x)+\sin 3x+\cos 3x=3\sqrt{2}(2+\sin 2x)$

2) $4\sin^2\frac{x}{2}-\sqrt{3}\cos 2x=1+2\cos^2(x-\frac{3\pi}{4})$

3) $3\cot^2 x+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

4) $\frac{1}{\tan x+\cot 2x}=\frac{\sqrt{2}\left(\cos x-\sin x\right)}{\cot x-1}$

5) $2+\sqrt{3}\left(\sin 2x-3\sin x\right)=\cos 2x+3\cos x$

6) $\frac{1+\sin 2x-\cos 2x}{1+\tan^2 x}=\cos x(\sin 2x+2\cos^2 x)$

7) $\tan^2 x+\left(1+\tan^2 x\right)\left(2-3\sin x\right)-1=0$

8) $\cos\left(x-\frac{\pi}{4}\right)+\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{3}\cos 2x-1$

9) $\cos 2x+5=2\sqrt{2}(2-\cos x)\sin(x-\frac{\pi}{4})$

10) $\frac{\cos^3 x-\cos^2 x}{\sin x+\cos x}=2\left(1+\sin x\right).$

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Bài giải

1) $2(\sin x-\cos x)+\sin 3x+\cos 3x=3\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 2(\sin x-\cos x)+3\sin x-4\sin^3 x+4\cos^3 x-3\cos x=3\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow 5(\sin x-\cos x)-4(\sin x-\cos x)(1+\sin x\cos x)=3\sqrt{2}(2+\sin 2x)$

$\Leftrightarrow (\sin x-\cos x)(1-4\sin x\cos x)=3\sqrt{2}(2+\sin 2x)$ (1)

+ Đặt $t=\sin x-\cos x$ $-\sqrt{2}\le t\le\sqrt{2}$ thì $t^2=1-\sin 2x$

+ (1) trở thành $t\left[1+2(t^2-1)\right]=3\sqrt{2}(3-t^2)$

$\Leftrightarrow 2t^3+3\sqrt{2}t^2-t-9\sqrt{2}=0$

$\Leftrightarrow (t-\sqrt{2})(2t^2+5\sqrt{2}t+9)=0\Leftrightarrow$ t = $\sqrt{2}$

+ $\sin x-\cos x=\sqrt{2}\Leftrightarrow\sin(x-\frac{\pi}{4})=1\Leftrightarrow x=\frac{3\pi}{4}+k2\pi$

2) $4\sin^2\frac{x}{2}-\sqrt{3}\cos 2x=1+2\cos^2(x-\frac{3\pi}{4})$

Ta có: $4\sin^2\frac{x}{2}-\sqrt{3}\cos 2x=1+2\cos^2(x-\frac{3\pi}{4})\Leftrightarrow 2(1-\cos x)-\sqrt{3}\cos 2x=1+1+\cos(2x-\frac{3\pi}{2})$

$\Leftrightarrow 2(1-\cos x)-\sqrt{3}\cos 2x=2-\sin 2x\Leftrightarrow\sqrt{3}\cos 2x-\sin 2x=-2\cos x$

$$\Leftrightarrow \frac{\sqrt{3}}{2}cos2x - \frac{1}{2}\sin 2x = -\cos x \Leftrightarrow cos(2x + \frac{\pi}{6}) = cos(\pi - x)$$

$$\Leftrightarrow \left[\begin{array}{l} 2x + \frac{\pi}{6} = \pi - x + k2\pi \\ 2x + \frac{\pi}{6} = x - \pi + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{5\pi}{18} + k\frac{2\pi}{3} \\ x = -\frac{7\pi}{6} + k2\pi \end{array}\right., k \in Z$$

3) $3\cot^2 x + 2\sqrt{2}\sin^2 x = (2 + 3\sqrt{2})\cos x$

Điều kiện : $x \neq k\pi$

$$3\cos x\left(\frac{\cos x}{\sin^2 x} - \sqrt{2}\right) = 2(\cos x - \sqrt{2}\sin^2 x)$$

$$\Leftrightarrow (\cos x - \sqrt{2}\sin^2 x)(3\cos x - 2\sin^2 x) = 0 \Leftrightarrow \left[\begin{array}{l} \sqrt{2}\cos^2 x + \cos x - \sqrt{2} = 0 \\ 2\cos^2 x + 3\cos x - 2 = 0 \end{array}\right.$$

$$\Leftrightarrow \cos x = -\sqrt{2}\ (loai); \cos x = \frac{\sqrt{2}}{2}; \cos x = -2\ (loai); \cos x = \frac{1}{2}$$

$$x = \pm\frac{\pi}{4} + k2\pi \ \ \& \ \ x = \pm\frac{\pi}{3} + k2\pi$$

4) $\dfrac{1}{\tan x + \cot 2x} = \dfrac{\sqrt{2}\left(\cos x - \sin x\right)}{\cot x - 1}$

Điều kiện: $\begin{cases} \cos x.\sin 2x.\sin x.\left(\tan x + \cot 2x\right) \neq 0 \\ \cot x \neq 1 \end{cases}$

Từ (1) ta có: $\dfrac{1}{\dfrac{\sin x}{\cos x} + \dfrac{\cos 2x}{\sin 2x}} = \dfrac{\sqrt{2}\left(\cos x - \sin x\right)}{\dfrac{\cos x}{\sin x} - 1} \Leftrightarrow \dfrac{\cos x.\sin 2x}{\cos x} = \sqrt{2}\sin x \Leftrightarrow 2\sin x.\cos x = \sqrt{2}\sin x$

$$\Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{4} + k2\pi \\ x = -\frac{\pi}{4} + k2\pi \end{array}\right. \left(k \in \mathbb{Z}\right)$$

Giao với điều kiện, ta được họ nghiệm của phương trình đã cho là $x = -\dfrac{\pi}{4} + k2\pi\ \left(k \in \mathbb{Z}\right)$

5) $2 + \sqrt{3}\left(\sin 2x - 3\sin x\right) = \cos 2x + 3\cos x$

Phương trình đã cho tương đương với:

$$1 + \frac{\sqrt{3}}{2}.\sin 2x - \frac{1}{2}\cos 2x - 3\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) = 0 \Leftrightarrow 1 - \cos\left(2x + \frac{\pi}{3}\right) - 3\sin\left(x + \frac{\pi}{6}\right) = 0$$

$\Leftrightarrow 2\sin^2\left(x+\frac{\pi}{6}\right)-3\sin\left(x+\frac{\pi}{6}\right)=0 \Leftrightarrow \sin\left(x+\frac{\pi}{6}\right)=0;\sin\left(x+\frac{\pi}{6}\right)=\frac{3}{2}$ (loai)

Với $\sin\left(x+\frac{\pi}{6}\right)=0 \Rightarrow x=-\frac{\pi}{6}+k\pi, k\in\mathbb{Z}.$

7) $\frac{1+\sin 2x-\cos 2x}{1+\tan^2 x}=\cos x(\sin 2x+2\cos^2 x)$

Điều kiện: cosx ≠ 0.

Biến đổi PT về:

$\cos^2x(1 + \sin2x - \cos2x) = \cos^2x\,(2\sin x + 2\cos x)$

$\Leftrightarrow$ 1 + sin2x – cos2x = 2(sinx + cosx) (vì cosx ≠ 0)

$\Leftrightarrow (\sin x + \cos x)^2 - (\cos^2x - \sin^2x) - 2(\sin x + \cos x) = 0$

$\Leftrightarrow$ (sinx + cosx)[sinx + cosx – (cosx – sinx) – 2] = 0

$\Leftrightarrow$ (sinx + cosx)(2sinx – 2) = 0 $\Leftrightarrow$ sinx + cosx = 0 hoặc 2sinx – 2 = 0

$\Leftrightarrow$ tanx = – 1 hoặc sinx = 1 (không thỏa cosx = 0) $\Leftrightarrow$ x = $-\frac{\pi}{4}+k\pi$, ($k\in\mathbb{Z}$)

8) $\tan^2 x+\left(1+\tan^2 x\right)\left(2-3\sin x\right)-1=0$

Điều kiện $\cos x\neq 0$

Phương trình viết lại $2-3\sin x=\frac{1-\tan^2 x}{1+\tan^2 x}$

$\Leftrightarrow 2-3\sin x=\cos 2x \Leftrightarrow 2\sin^2 x-3\sin x+1=0 \Leftrightarrow \sin x=1\,;\sin x=\frac{1}{2}$

So sánh đ/k chọn $\sin x=\frac{1}{2}\Leftrightarrow x=\frac{\pi}{6}+k2\pi;\ x=\frac{5\pi}{6}+k2\pi\ \left(k\in\mathbb{Z}\right)$

9) $\cos\left(x-\frac{\pi}{4}\right)+\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{3}\cos 2x-1 \Leftrightarrow 2\cos x.\cos\frac{\pi}{4}=\frac{1}{3}\left(2\cos^2 x-1\right)-1$

$\Leftrightarrow 3\sqrt{2}\cos x=2\cos^2 x-4 \Leftrightarrow 2\cos^2 x-3\sqrt{2}\cos x-4=0$

$\Leftrightarrow (\cos x-2\sqrt{2})(\cos x+\frac{\sqrt{2}}{2})=0 \Leftrightarrow \cos x=-\frac{\sqrt{2}}{2} \Leftrightarrow x=\pm\frac{3\pi}{4}+2k\pi$.

10) $\cos 2x+5=2\sqrt{2}(2-\cos x)\sin(x-\frac{\pi}{4})$

Phương trình $\Leftrightarrow (\cos x-\sin x)^2 - 4(\cos x-\sin x) - 5 = 0 \qquad \Leftrightarrow \left[\begin{array}{l}\cos x-\sin x=-1\\ \cos x-\sin x=5 \quad (loai\ vi\ \cos x-\sin x\le\sqrt{2})\end{array}\right.$

$$\Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1 \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\sin\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=\pi+k2\pi\end{array}\right.(k\in Z)$$

10) $\dfrac{\cos^3 x-\cos^2 x}{\sin x+\cos x}=2\left(1+\sin x\right).$

ĐK: $\sin x+\cos x\ne 0$

Khi đó $PT \Leftrightarrow \left(1-\sin^2 x\right)\left(\cos x-1\right)=2\left(1+\sin x\right)\left(\sin x+\cos x\right)$

$$\Leftrightarrow \left(1+\sin x\right)\left(1+\cos x+\sin x+\sin x.\cos x\right)=0 \Leftrightarrow \left(1+\sin x\right)\left(1+\cos x\right)\left(1+\sin x\right)=0$$

$\Leftrightarrow \left[\begin{array}{l}\sin x=-1\\ \cos x=-1\end{array}\right.$ (thoả mãn điều kiện) $\Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{2}+k2\pi\\ x=\pi+m2\pi\end{array}\right. \quad \left(k,m\in\mathbb{Z}\right)$

Vậy phương trình đã cho có nghiệm là: $x=-\dfrac{\pi}{2}+k2\pi$ và $x=\pi+m2\pi \ \left(k,m\in\mathbb{Z}\right)$

HT 8.Giải các phương trình sau:

1) $\dfrac{4\sin^4 x+4\cos^4(x-\frac{\pi}{4})-1}{\cos 2x}=2$

2) $\dfrac{4\sin x.\sin(x+\frac{\pi}{3})+5\sqrt{3}\sin x+3(\cos x+2)}{1-2\cos x}=1$

3) $\dfrac{\cos^2 x.\left(\cos x-1\right)}{\sin x+\cos x}=2\left(1+\sin x\right)$

4) $\dfrac{\left(\sin x+\cos x\right)^2-2\sin^2 x}{1+\cot^2 x}=\dfrac{\sqrt{2}}{2}\left[\sin\left(\frac{\pi}{4}-x\right)-\sin\left(\frac{\pi}{4}-3x\right)\right]$

5) $\dfrac{\sin 2x}{\sin x+\cos x}+\dfrac{1}{\sqrt{2}.\tan x}=2\cos x \quad (1)$

6) $\dfrac{1}{\cos^2 x}-(\cos x+\sin x.\tan\frac{x}{2})=\dfrac{\sin(x-\frac{\pi}{6})+\cos(\frac{\pi}{3}-x)}{\cos x}$

7) $2\cos^2 x+2\sqrt{3}\sin x\cos x+1=3(\sin x+\sqrt{3}\cos x)$

8) $\dfrac{(1-\sin x+\sqrt{2}\cos 2x)\sin(x+\frac{\pi}{4})}{1+\cot x}=\dfrac{1}{\sqrt{2}}\sin x(\cos x+1)$

9) $\dfrac{\left(\sin x+\cos x\right)^2-2\sin^2 x}{1+\cot^2 x}=\dfrac{\sqrt{2}}{2}\left(\sin\left(\frac{\pi}{4}-x\right)-\sin\left(\frac{\pi}{4}-3x\right)\right)$

10) $\sin x\cos 2x+\cos^2 x\left(\tan^2 x-1\right)+2\sin^3 x=0$

Bài giải

1) $\dfrac{4\sin^4 x + 4\cos^4(x - \frac{\pi}{4}) - 1}{\cos 2x} = 2$ (1)

ĐK: $\cos 2x \neq 0 \Leftrightarrow x \neq \frac{\pi}{4} + k\frac{\pi}{2} (k \in \mathbb{Z})$

$(1) \Leftrightarrow (1 - \cos 2x)^2 + \left(1 + \cos(2x - \frac{\pi}{2})\right)^2 - 1 = 2\cos 2x$

$\Leftrightarrow (1 - \cos 2x)^2 + (1 + \sin 2x)^2 - 1 = 2\cos 2x$

$\Leftrightarrow 2 - 2\cos 2x + 2\sin 2x = 2\cos 2x \Leftrightarrow 2\cos 2x - \sin 2x = 1$

$\Leftrightarrow 2(\cos^2 x - \sin^2 x) - (\cos x + \sin x)^2 = 0$

$\Leftrightarrow (\cos x + \sin x)(\cos x - 3\sin x) = 0 \Leftrightarrow \left[\begin{array}{l} \cos x + \sin x = 0 \\ \cos x - 3\sin x = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{4} + k\pi \\ x = \arctan 3 + k\pi \end{array}\right. (k \in \mathbb{Z})$

Kết hợp với điều kiện phương trình đã cho có nghiệm là $x = \arctan 3 + k\pi \quad (k \in \mathbb{Z})$

2) $\dfrac{4\sin x.\sin(x + \frac{\pi}{3}) + 5\sqrt{3}\sin x + 3(\cos x + 2)}{1 - 2\cos x} = 1$

Điều kiện: $x \neq \pm\frac{\pi}{3} + k2\pi$

$PT \Leftrightarrow 1 - 2.\cos(2x + \frac{\pi}{3}) + 5(\sqrt{3}\sin x + \cos x) + 5 = 0 \Leftrightarrow 4.\sin^2(x + \frac{\pi}{6}) + 10\sin(x + \frac{\pi}{6}) + 4 = 0$

$\Leftrightarrow \left[\begin{array}{l} \sin(x + \frac{\pi}{6}) = -1/2 \\ \sin(x + \frac{\pi}{6}) = -2 \ (VN) \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{3} + k2\pi \quad \text{(L)} \\ x = \pi + k2\pi \end{array}\right.$

Vậy, $S = \left\{\pi + k2\pi\right\}$

3) $\dfrac{\cos^2 x.\left(\cos x - 1\right)}{\sin x + \cos x} = 2\left(1 + \sin x\right)$

ĐK: $x \neq -\frac{\pi}{4} + k\pi$.

PT $\Leftrightarrow (1 + \sin x)(1 - \sin x)(\cos x - 1) = 2(1 + \sin x)(\sin x + \cos x)$

$\Leftrightarrow \left[\begin{array}{l} 1 + \sin x = 0 \\ \sin x + \cos x + \sin x\cos x + 1 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} 1 + \sin x = 0 \\ \left(1 + \sin x\right)\left(\cos x + 1\right) = 0 \end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} x = -\frac{\pi}{2} + k2\pi \\ x = \pi + k2\pi \end{array}\right.$ (Thoả mãn điều kiện)

4) $\dfrac{\left(\sin x+\cos x\right)^2-2\sin^2 x}{1+\cot^2 x}=\dfrac{\sqrt{2}}{2}\left[\sin\left(\dfrac{\pi}{4}-x\right)-\sin\left(\dfrac{\pi}{4}-3x\right)\right]$.

Điều kiện xác định $\sin x\neq 0$ hay $x\neq k\pi;\ k\in\mathbb{Z}$.

Phương trình đã cho tương đương với

$$\left(\cos 2x+\sin 2x\right)\sin^2 x=\sqrt{2}\cos\left(\frac{\pi}{4}-2x\right)\sin x\Leftrightarrow\cos\left(\frac{\pi}{4}-2x\right)\left(\sin x-1\right)=0$$

$$\Leftrightarrow\left[\begin{array}{l}\cos\left(\dfrac{\pi}{4}-2x\right)=0\\ \sin x-1=0\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{3\pi}{8}+\dfrac{k\pi}{2}\\ x=\dfrac{\pi}{2}+m2\pi\end{array}\right.\quad\left(k,m\in Z\right)$$

So với điều kiện nghiệm của phương trình là $x=\dfrac{3\pi}{8}+\dfrac{k\pi}{2};\ x=\dfrac{\pi}{2}+m2\pi;\ \left(k,m\in Z\right)$

5) $\dfrac{\sin 2x}{\sin x+\cos x}+\dfrac{1}{\sqrt{2}.\tan x}=2\cos x\ \ (1)$

Điều kiện: $\sin x\neq 0,\ \cos x\neq 0,\sin x+\cos x\neq 0.$

$$(1)\Leftrightarrow\frac{\cos x}{\sqrt{2}\sin x}+\frac{2\sin x\cos x}{\sin x+\cos x}-2\cos x=0$$

$$\Leftrightarrow\frac{\cos x}{\sqrt{2}\sin x}-\frac{2\cos^2 x}{\sin x+\cos x}=0\Leftrightarrow\cos x\left(\sin(x+\frac{\pi}{4})-\sin 2x\right)=0$$

+) $\cos x=0\Leftrightarrow x=\dfrac{\pi}{2}+k\pi,\ \ k\in Z.$

+) $\sin 2x=\sin(x+\dfrac{\pi}{4})\Leftrightarrow\left[\begin{array}{l}2x=x+\dfrac{\pi}{4}+m2\pi\\ 2x=\pi-x-\dfrac{\pi}{4}+n2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\dfrac{\pi}{4}+m2\pi\\ x=\dfrac{\pi}{4}+\dfrac{n2\pi}{3}\end{array}\right.m,\ n\in Z\Leftrightarrow x=\dfrac{\pi}{4}+\dfrac{t2\pi}{3},\ \ t\in Z.$

So sánh điều kiện, nghiệm của phương trình: $x=\dfrac{\pi}{2}+k\pi;\ \ x=\dfrac{\pi}{4}+\dfrac{t2\pi}{3},\ \ k,\ t\in Z.$

6) $\dfrac{1}{\cos^2 x}-(\cos x+\sin x.\tan\dfrac{x}{2})=\dfrac{\sin(x-\dfrac{\pi}{6})+\cos(\dfrac{\pi}{3}-x)}{\cos x}$

Điều kiện $\left\{\begin{array}{l}\cos x\neq 0\\ \cos\dfrac{x}{2}\neq 0\end{array}\right.$.

Phương trình $\Leftrightarrow \dfrac{1}{\cos^2 x} - (\cos x + 2\sin^2\dfrac{x}{2}) = \dfrac{\cos(\dfrac{2\pi}{3} - x) + \cos(\dfrac{\pi}{3} - x)}{\cos x}$

$\Leftrightarrow \dfrac{1}{\cos^2 x} - (\cos x + 1 - \cos x) = \dfrac{2\cos(\dfrac{\pi}{2} - x)\cos\dfrac{\pi}{6}}{\cos x} \Leftrightarrow \dfrac{1}{\cos^2 x} - 1 = \dfrac{\sqrt{3}\sin x}{\cos x} \Leftrightarrow \tan^2 x = \sqrt{3}\tan x$

$\tan^2 x - \sqrt{3}\tan x = 0 \Leftrightarrow \left[\begin{array}{l} \tan x = 0 \\ \tan x = \sqrt{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \dfrac{\pi}{3} + k\pi \end{array}\right. (k \in Z)$

Đối chiếu điều kiện ta thấy nghiệm của phương trình là $\left[\begin{array}{l} x = 2l\pi \\ x = \dfrac{\pi}{3} + l\pi \end{array}\right. (l \in Z)$

7) Giải phương trình $2\cos^2 x + 2\sqrt{3}\sin x\cos x + 1 = 3(\sin x + \sqrt{3}\cos x)$.

$2\cos^2 x + 2\sqrt{3}\sin x\cos x + 1 = 3(\sin x + \sqrt{3}\cos x) \Leftrightarrow (\sin x + \sqrt{3}\cos x)^2 - 3(\sin x + \sqrt{3}\cos x) = 0$

$\Leftrightarrow \sin x + \sqrt{3}\cos x = 0 \vee \sin x + \sqrt{3}\cos x = 3$ (1)

Phương trình $\sin x + \sqrt{3}\cos x = 3$ vô nghiệm vì $1^2 + (\sqrt{3})^2 < 3^2$

Nên (1) $\Leftrightarrow \tan x = -\sqrt{3} \Leftrightarrow x = -\dfrac{\pi}{3} + k\pi$ ($k \in \mathbb{Z}$). Vậy, PT có nghiệm là: $x = -\dfrac{\pi}{3} + k\pi$ ($k \in \mathbb{Z}$).

8) $\dfrac{(1 - \sin x + \sqrt{2}\cos 2x)\sin(x + \dfrac{\pi}{4})}{1 + \cot x} = \dfrac{1}{\sqrt{2}}\sin x(\cos x + 1)$

Đk : $\begin{cases} \sin x \neq 0 \\ \cot x \neq -1 \end{cases}$

pt $\Leftrightarrow \dfrac{(1 - \sin x + \sqrt{2}\cos 2x)(\sin x + \cos x)}{\sqrt{2}.\dfrac{\sin x + \cos x}{\sin x}} = \dfrac{1}{\sqrt{2}}.\sin x.(\cos x + 1)$

$\Leftrightarrow$ 1 – sinx + $\sqrt{2}$ cos2x = cosx + 1 $\Leftrightarrow$ sinx + cosx = $\sqrt{2}$ cos2x

$\Leftrightarrow$ sinx + cosx = $\sqrt{2}$ (cosx + sinx)(cosx – sinx) $\Leftrightarrow$ $\sqrt{2}$ (cosx – sinx) = 1

$\Leftrightarrow$ 2cos$\left(x + \dfrac{\pi}{4}\right)$ = 1 $\Leftrightarrow$ cos$\left(x + \dfrac{\pi}{4}\right)$ = cos$\dfrac{\pi}{3}$

Kết hợp đk => nghiệm phương trình : x = $\dfrac{\pi}{12} + k2\pi$ hoặc x = $-\dfrac{7\pi}{12} + k2\pi$

.

9) $\dfrac{\left(\sin x+\cos x\right)^2-2\sin^2 x}{1+\cot^2 x}=\dfrac{\sqrt{2}}{2}\left(\sin\left(\dfrac{\pi}{4}-x\right)-\sin\left(\dfrac{\pi}{4}-3x\right)\right)$

Điều kiện: $\sin x\neq 0$ (*). Khi đó:

Phương trình đã cho tương đương với: $\left(\sin 2x+\cos 2x\right).\sin^2 x=\sqrt{2}\cos\left(\dfrac{\pi}{4}-2x\right).\sin x$

$\Leftrightarrow \cos\left(2x-\dfrac{\pi}{4}\right).\sin x=\cos\left(2x-\dfrac{\pi}{4}\right)\Leftrightarrow\left(\sin x-1\right).\cos\left(2x-\dfrac{\pi}{4}\right)=0$

+ $\sin x=1\Leftrightarrow x=\dfrac{\pi}{2}+k2\pi\quad\left(k\in\mathbb{Z}\right)$, thỏa (*) + $\cos\left(2x-\dfrac{\pi}{4}\right)=0\Leftrightarrow x=\dfrac{3\pi}{8}+\dfrac{k\pi}{2}\ \left(k\in\mathbb{Z}\right)$, thỏa (*)

Vậy, phương trình có nghiệm: $x=\dfrac{\pi}{2}+k2\pi;x=\dfrac{3\pi}{8}+\ \dfrac{k\pi}{2}\ \left(k\in\mathbb{Z}\right)$.

10) $\sin x\cos 2x+\cos^2 x\left(\tan^2 x-1\right)+2\sin^3 x=0$

Điều kiện $\cos x\neq 0$

$\sin x\cos 2x+\cos^2 x\left(\tan^2 x-1\right)+2\sin^3 x=0\ \Leftrightarrow\ \sin x\left(1-2\sin^2 x\right)+2\sin^2 x-1+2\sin^3 x=0$

$\Leftrightarrow 2\sin^2 x+\sin x-1=0\Leftrightarrow\left[\begin{array}{l}\sin x=-1\\ \sin x=\dfrac{1}{2}\end{array}\right.\Leftrightarrow x=-\dfrac{\pi}{2}+k2\pi;x=\dfrac{\pi}{6}+k2\pi;x=\dfrac{5\pi}{6}+k2\pi$.

Kết hợp điều kiện, phương trình có nghiệm $S=\left\{\dfrac{\pi}{6}+k2\pi;\dfrac{5\pi}{6}+k2\pi\right\}$

HT 9.Giải các phương trình sau:

1) $\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)-\sin x-3\cos x+2=0$

2) $\dfrac{2\cos^2 x+\sqrt{3}\sin 2x+3}{2\cos^2 x.\sin\left(x+\dfrac{\pi}{3}\right)}=3\left(\tan^2 x+1\right)$

3) $\dfrac{3-4\cos 2x-8\sin^4 x}{\sin 2x+\cos 2x}=\dfrac{1}{\sin 2x}$

4) $\sin^2 x+\sin^2\left(\dfrac{\pi}{3}-x\right)+\sin^2\left(\dfrac{\pi}{3}+x\right)=2\sqrt{3}\sin\left(x+\dfrac{\pi}{6}\right).\cos x-\dfrac{\sqrt{3}}{2}$

5) $\dfrac{3\cot^2 x+2\sqrt{2}\sin^2 x-\left(2+3\sqrt{2}\right)\cos x}{2\sin x+\sqrt{3}}=0$

6) $\dfrac{\cos x(\cos x+2\sin x)+3\sin x(\sin x+\sqrt{2})}{\sin 2x-1}=1$

7) $\dfrac{1}{\sqrt{2}}\cot x+\dfrac{\sin 2x}{\sin x+\cos x}=2\sin(x+\dfrac{\pi}{2})$

8) $\dfrac{2\cos^2 x-2\sqrt{3}\sin x\cos x+1}{2\cos 2x}=\sqrt{3}\cos x-\sin x$

9) $\sin^2\left(\dfrac{x}{2}+\dfrac{7\pi}{4}\right)\tan^2(3\pi-x)-\cos^2\dfrac{x}{2}=0.$

10) $\dfrac{\sin^3 x\sin 3x+\cos^3 x.\cos 3x}{\tan\left(x-\dfrac{\pi}{6}\right)\tan\left(x+\dfrac{\pi}{3}\right)}=-\dfrac{1}{8}$

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Bài giải

1) $\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)-\sin x-3\cos x+2=0$

$\sqrt{2}\sin\left(2x+\dfrac{\pi}{4}\right)-\sin x-3\cos x+2=0\Leftrightarrow \sin 2x+\cos 2x-\sin x-3\cos x+2=0$

$\Leftrightarrow 2\sin x\cos x-\sin x+2\cos^2 x-3\cos x+1=0\Leftrightarrow \sin x(2\cos x-1)+(\cos x-1)(2\cos x-1)=0$

$\Leftrightarrow (2\cos x-1)(\sin x+\cos x-1)=0\Leftrightarrow \cos x=\dfrac{1}{2},\sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}$

Nghiệm phương trình: $x=\pm\dfrac{\pi}{3}+k2\pi$, $x=k2\pi$, $x=\dfrac{\pi}{2}+k2\pi$

2) $\dfrac{2\cos^2 x+\sqrt{3}\sin 2x+3}{2\cos^2 x.\sin\left(x+\dfrac{\pi}{3}\right)}=3(\tan^2 x+1).$

Điều kiện: $\begin{cases}\cos x\neq 0\\ \sin\left(x+\dfrac{\pi}{3}\right)\neq 0\end{cases}\Leftrightarrow\begin{cases}x\neq\dfrac{\pi}{2}+k\pi\\ x\neq-\dfrac{\pi}{3}+k\pi\end{cases}(k\in Z)$ (*).

Khi đó:Phương trình đã cho tương đương với: $\cos 2x+\sqrt{3}\sin 2x+4=2\cos^2 x\sin\left(x+\dfrac{\pi}{3}\right)\dfrac{3}{\cos^2 x}$

$\Leftrightarrow \cos 2x.\cos\dfrac{\pi}{3}+\sin 2x.\sin\dfrac{\pi}{3}+2=3\sin\left(x+\dfrac{\pi}{3}\right)$

$\Leftrightarrow \cos\left(2x-\dfrac{\pi}{3}\right)-3\sin\left(x+\dfrac{\pi}{3}\right)+2=0\Leftrightarrow 2\cos^2\left(x-\dfrac{\pi}{6}\right)-3\cos\left(x-\dfrac{\pi}{6}\right)+1=0$

$\Leftrightarrow \cos\left(x-\dfrac{\pi}{6}\right)=1,\cos\left(x-\dfrac{\pi}{6}\right)=\dfrac{1}{2}$

Với $\cos\left(x-\dfrac{\pi}{6}\right)=1\Leftrightarrow x-\dfrac{\pi}{6}=k2\pi\Leftrightarrow x=\dfrac{\pi}{6}+k2\pi\ \left(k\in\mathbb{Z}\right)$, thỏa (*)

Với $\cos\left(x-\dfrac{\pi}{6}\right)=\dfrac{1}{2}\Leftrightarrow\left[\begin{array}{l}x-\dfrac{\pi}{6}=\dfrac{\pi}{3}+k2\pi\\ x-\dfrac{\pi}{6}=-\dfrac{\pi}{3}+k2\pi\end{array}\right.\Rightarrow x=-\dfrac{\pi}{6}+k2\pi\ \left(k\in\mathbb{Z}\right)$, thỏa (*)

Vậy, phương trình có nghiệm: $x=\pm\dfrac{\pi}{6}+k2\pi\ \left(k\in\mathbb{Z}\right)$.

3) $\dfrac{3-4\cos2x-8\sin^4x}{\sin2x+\cos2x}=\dfrac{1}{\sin2x}$

Điều kiện: $\begin{cases}\sin2x+\cos2x\neq0\\ \sin2x\neq0\end{cases}\Leftrightarrow\begin{cases}x\neq-\dfrac{\pi}{8}+l\dfrac{\pi}{2}\\ x\neq l\dfrac{\pi}{2}\end{cases}\left(l\in\mathbb{Z}\right)$

Ta có: $8\sin^4x=8\left(\dfrac{1-\cos2x}{2}\right)^2=\cdots=3-4\cos2x+\cos4x$

Phương trình $\Leftrightarrow\dfrac{3-4\cos2x-\left(3-4\cos2x+\cos4x\right)}{\sin2x+\cos2x}=\dfrac{1}{\sin2x}$

$\Leftrightarrow\dfrac{-\cos4x}{\sin2x+\cos2x}=\dfrac{1}{\sin2x}\left(do\sin2x+\cos2x\neq0,\sin2x\neq0\right)$

$\Leftrightarrow-\left(\cos2x-\sin2x\right)=\dfrac{1}{\sin2x}\Leftrightarrow\cos2x\left(\sin2x+\cos2x\right)=0$

$\Leftrightarrow\cos2x=0\vee\sin2x+\cos2x=0\left(loai\right)\Leftrightarrow2x=\dfrac{\pi}{2}+k\pi\Leftrightarrow x=\dfrac{\pi}{4}+k\dfrac{\pi}{2}\left(k\in\mathbb{Z}\right)$

Vậy nghiệm của phương trình: $x=\dfrac{\pi}{4}+k\dfrac{\pi}{2}\left(k\in\mathbb{Z}\right)$

4) $\sin^2x+\sin^2\left(\dfrac{\pi}{3}-x\right)+\sin^2\left(\dfrac{\pi}{3}+x\right)=2\sqrt{3}\sin\left(x+\dfrac{\pi}{6}\right).\cos x-\dfrac{\sqrt{3}}{2}$

Ta c� $\sin^2x+\sin^2\left(\dfrac{\pi}{3}-x\right)+\sin^2\left(\dfrac{\pi}{3}+x\right)=2\sqrt{3}\sin\left(x+\dfrac{\pi}{6}\right).\cos x-\dfrac{\sqrt{3}}{2}$

$\Leftrightarrow\dfrac{1-\cos2x+1-\cos\left(\dfrac{2\pi}{3}-2x\right)+1+\cos\left(\dfrac{2\pi}{3}-2x\right)}{2}=\sqrt{3}\left(\sqrt{3}\sin x+\cos x\right)\cos x-\dfrac{\sqrt{3}}{2}$

$$\Leftrightarrow \frac{3-\cos 2x-2\cos\frac{2\pi}{3}\cos 2x}{2}=3\sin x\cos x+\sqrt{3}\cos^2 x-\frac{\sqrt{3}}{2}\Leftrightarrow 3=3\sin 2x+\sqrt{3}\left(2\cos^2 x-1\right)$$

$$\Leftrightarrow \sqrt{3}\sin 2x+\cos 2x=\sqrt{3}\Leftrightarrow \sin 2x.\frac{\sqrt{3}}{2}+\cos 2x.\frac{1}{2}=\frac{\sqrt{3}}{2}\Leftrightarrow \sin\left(2x+\frac{\pi}{6}\right)=\sin\frac{\pi}{3}$$

$$\Leftrightarrow \left[\begin{array}{l}2x+\frac{\pi}{6}=\frac{\pi}{3}+k2\pi\\2x+\frac{\pi}{6}=\pi-\frac{\pi}{3}+k2\pi\end{array}\right.\Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{12}+k\pi\\x=\frac{\pi}{4}+k\pi\end{array}\right.\left(k\in\mathbb{Z}\right)$$

5) $\dfrac{3\cot^2 x+2\sqrt{2}\sin^2 x-\left(2+3\sqrt{2}\right)\cos x}{2\sin x+\sqrt{3}}=0$

Điều kiện: cosx $\neq 0$, $\sin x\neq -\frac{\sqrt{3}}{2}$

Khi đó pt đã cho $\Leftrightarrow \left(3\cot^2 x-3\sqrt{2}\cos x\right)+\left(2\sqrt{2}\sin^2 x-2\cos x\right)=0$

$$\Leftrightarrow 3\cos x\left(\frac{\cos x}{\sin^2 x}-\sqrt{2}\right)+2\left(\sqrt{2}\sin^2 x-\cos x\right)=0\Leftrightarrow \left(3\cos x-2\sin^2 x\right)\left(\sqrt{2}\sin^2 x-\cos x\right)=0$$

$+)\cos x-\sqrt{2}\sin^2 x=0\Leftrightarrow \sqrt{2}\cos^2 x+\cos x-\sqrt{2}=0$

$\cos x=-\sqrt{2}\left(L\right),\cos x=\frac{1}{\sqrt{2}}\Leftrightarrow x=\pm\frac{\pi}{4}+k2\pi$

$+)3\cos x-2\sin^2 x=0\Leftrightarrow 2\cos^2 x+3\cos x-2=0$

$\Leftrightarrow \cos x=-2\left(L\right),\cos x=\frac{1}{2}\Leftrightarrow x=\pm\frac{\pi}{3}+k2\pi$.

Đối chiếu với đ/k bài toán thì pt chỉ có 3 họ nghiệm:. $x=\pm\frac{\pi}{4}+k2\pi,x=\frac{\pi}{3}+k2\pi,k\in\mathbb{Z}$

6) $\dfrac{\cos x(\cos x+2\sin x)+3\sin x(\sin x+\sqrt{2})}{\sin 2x-1}=1$

Điều kiện: sin2x ≠ 1.

Pt $\Leftrightarrow$ $\cos^2 x+2\sin x\cos x+3\sin^2 x+3\sqrt{2}\sin x=\sin 2x-1\Leftrightarrow 2\sin^2 x+3\sqrt{2}\sin x+2=0$

$$\Leftrightarrow \left[\begin{array}{l}\sin x=\frac{-\sqrt{2}}{2}\\\sin x=-\sqrt{2}\end{array}\right.\Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{4}+k2\pi\\x=\frac{5\pi}{4}+k2\pi\end{array}\right.$$

Đối chiếu điều kiện ta có nghiệm: $x=-\frac{\pi}{4}+k2\pi$.

7) $\frac{1}{\sqrt{2}}\cot x+\frac{\sin 2x}{\sin x+\cos x}=2\sin(x+\frac{\pi}{2})$

Điều kiện: $\sin x\neq 0,\ \sin x+\cos x\neq 0.$

PT: $\frac{\cos x}{\sqrt{2}\sin x}+\frac{2\sin x\cos x}{\sin x+\cos x}-2\cos x=0\Leftrightarrow\frac{\cos x}{\sqrt{2}\sin x}-\frac{2\cos^2 x}{\sin x+\cos x}=0\Leftrightarrow\cos x\left(\sin(x+\frac{\pi}{4})-\sin 2x\right)=0$

+) $\cos x=0\Leftrightarrow x=\frac{\pi}{2}+k\pi,\quad k\in Z.$

+) $\sin 2x=\sin(x+\frac{\pi}{4})\Leftrightarrow\left[\begin{array}{l}2x=x+\frac{\pi}{4}+m2\pi\\2x=\pi-x-\frac{\pi}{4}+n2\pi\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\frac{\pi}{4}+m2\pi\\x=\frac{\pi}{4}+\frac{n2\pi}{3}\end{array}\right.\quad m,\ n\in Z\ \Leftrightarrow x=\frac{\pi}{4}+\frac{t2\pi}{3},\ t\in Z.$

Đối chiếu điều kiện ta có nghiệm của pt là : $x=\frac{\pi}{2}+k\pi$; $x=\frac{\pi}{4}+\frac{t2\pi}{3},\ k,\ t\in Z.$

8) $\frac{2\cos^2 x-2\sqrt{3}\sin x\cos x+1}{2\cos 2x}=\sqrt{3}\cos x-\sin x$

Điều kiện: cos2x $\neq$ 0 (*)

Pt đã cho $\Leftrightarrow\frac{3\cos^2 x-2\sqrt{3}\sin x\cos x+\sin^2 x}{2\cos 2x}=\sqrt{3}\cos x-\sin x$

$\Leftrightarrow(\sqrt{3}\cos x-\sin x)^2=2\cos 2x$ $(\sqrt{3}\cos x-\sin x)$

$\Leftrightarrow\left[\begin{array}{l}\sqrt{3}\cos x-\sin x=0\\2\cos 2x=\sqrt{3}\cos x-\sin x\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\tan x=\sqrt{3}\\\cos 2x=\cos(x+\frac{\pi}{6})\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=\frac{\pi}{3}+k\pi\\x=\frac{\pi}{6}+k2\pi,x=-\frac{\pi}{18}+k\frac{2\pi}{3}\end{array}\right.$

Các nghiệm đều TMĐK (*) nên phương trình đã cho có 3 họ nghiệm:

$$x=\frac{\pi}{3}+k\pi,\ x=\frac{\pi}{6}+k2\pi,\ x=-\frac{\pi}{18}+k\frac{2\pi}{3}\ (k\in Z)\ .$$

9) $\sin^2\left(\frac{x}{2}+\frac{7\pi}{4}\right)\tan^2(3\pi-x)-\cos^2\frac{x}{2}=0.$

Đ/k: $\cos x\neq 0$

Pt đã cho

$\Leftrightarrow \sin^2\left(\frac{x}{2}-\frac{\pi}{4}\right)\tan^2 x - cos^2\frac{x}{2} = 0 \Leftrightarrow \frac{1}{2}\left[1-\cos\left(x-\frac{\pi}{2}\right)\right]\frac{\sin^2 x}{\cos^2 x} - \frac{1}{2}\left(1+\cos x\right) = 0$

$\Leftrightarrow \left(1-\sin x\right)\left(1-\cos^2 x\right)-\left(1+\cos x\right)\left(1-\sin^2 x\right) = 0 \Leftrightarrow \left(1-\sin x\right)\left(1+\cos x\right)\left(\sin x+\cos x\right) = 0$

$\Leftrightarrow \left[\begin{array}{l}\sin x = 1 \quad loai\\ \cos x = -1\\ \tan x = -1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = \left(2k+1\right)\pi\\ x = -\frac{\pi}{4}+k\pi\end{array}\right. k \in Z$

10) $\frac{\sin^3 x\sin 3x + \cos^3 x.\cos 3x}{\tan\left(x-\frac{\pi}{6}\right)\tan\left(x+\frac{\pi}{3}\right)} = -\frac{1}{8}$

Điều kiện: $x \neq \frac{\pi}{6}+\frac{k\pi}{2}$

Ta có $\tan\left(x-\frac{\pi}{6}\right)\tan\left(x+\frac{\pi}{3}\right) = \tan\left(x-\frac{\pi}{6}\right)\cot\left(-x+\frac{\pi}{6}\right) = -1$

Phương trình tương đương với: $\sin^3 x\sin 3x + \cos^3 x\cos 3x = \frac{1}{8}$

$\Leftrightarrow \frac{1-\cos 2x}{2}.\frac{\cos 2x-\cos 4x}{2}+\frac{1+\cos 2x}{2}.\frac{\cos 2x+\cos 4x}{2} = \frac{1}{8}$

$\Leftrightarrow 2(\cos 2x - \cos 2x.\cos 4x) = \frac{1}{2}$

$\Leftrightarrow \cos^3 x = \frac{1}{8} \Leftrightarrow \cos 2x = \frac{1}{2}$

$x = -\frac{\pi}{6}+k\pi$ và $x = \frac{\pi}{6}+k\pi$ (loại)

Vậy : $x = -\frac{\pi}{6}+k\pi$

HT 10.Giải các phương trình sau:

1) $\sin 3x + \sin 2x + \sin x + 1 = \cos 3x + \cos 2x - \cos x.$

2) $(\tan x + 1)\sin^2 x + \cos 2x + 2 = 3(\cos x + \sin x)\sin x.$

3) $\sqrt{2}.\cos 5x - \sin(\pi + 2x) = \sin\left(\frac{5\pi}{2}+2x\right).\cot 3x.$

4) $\frac{6\sqrt{2}\sin^3 2x + 8\cos^3 x + 3\sqrt{2}\cos(\frac{17\pi}{2}-4x)\cos 2x}{\cos x} = 16$

5) $\frac{\sqrt{3}}{\cos^2 x}+\frac{4}{\sin 2x} = 2(\cot x + \sqrt{3})$

6) $\cos 2x + 2\sin x - 1 - 2\sin x\cos 2x = 0$

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Bài giải

1) $\sin 3x + \sin 2x + \sin x + 1 = \cos 3x + \cos 2x - \cos x.$

$\Leftrightarrow 2\sin 2x\cos x + 2\sin x\cos x + 2\sin^2 x = -2\sin 2x\cos x$

$\Leftrightarrow \sin 2x(\cos x + \sin x) + \sin x(\cos x + \sin x) = 0$

$\Leftrightarrow \sin x(2\cos x + 1)(\cos x + \sin x) = 0.$

Từ đó ta có các trường hợp sau

*) $\sin x = 0 \Leftrightarrow x = k\pi,\ k \in Z$

*) $2\cos x + 1 = 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow x = \pm\frac{2\pi}{3} + k2\pi,\ k \in Z$

*) $\cos x + \sin x = 0 \Leftrightarrow x = -\frac{\pi}{4} + k\pi,\ k \in Z$

Vậy phương trình đã cho có nghiệm $x = k\pi,\ x = \pm\frac{2\pi}{3} + k2\pi,\ x = -\frac{\pi}{4} + k\pi,\ k \in Z$

2) $(\tan x + 1)\sin^2 x + \cos 2x + 2 = 3(\cos x + \sin x)\sin x.$

Điều kiện: $\cos x \neq 0,$ hay $x \neq \frac{\pi}{2} + k\pi.$

Khi đó phương trình đã cho tương đương với

$(\tan x + 1)\sin^2 x + 1 - 2\sin^2 x + 2 = 3(\cos x + \sin x)\sin x$

$\Leftrightarrow (\tan x - 1)\sin^2 x + 3 = 3(\cos x - \sin x)\sin x + 6\sin^2 x$

$\Leftrightarrow (\tan x - 1)\sin^2 x + 3\cos 2x = 3(\cos x - \sin x)\sin x$

$\Leftrightarrow (\tan x - 1)\sin^2 x + 3(\cos x - \sin x)\cos x = 0$

$\Leftrightarrow (\sin x - \cos x)(\sin^2 x - 3\cos^2 x) = 0 \Leftrightarrow (\sin x - \cos x)(2\cos 2x + 1) = 0$

$\Leftrightarrow \sin x = \cos x, \cos 2x = -\frac{1}{2} \Leftrightarrow x = \frac{\pi}{4} + k\pi, x = \pm\frac{\pi}{3} + k\pi,\ k \in$�

Đối chiếu điều kiện ta có nghiệm $x = \frac{\pi}{4} + k\pi,\ x = \pm\frac{\pi}{3} + k\pi,\ k \in$�

3) $\sqrt{2}.\cos 5x - \sin(\pi + 2x) = \sin\left(\frac{5\pi}{2} + 2x\right).\cot 3x.$

ĐK: $\sin 3x \neq 0$

pt $\Leftrightarrow \sqrt{2}\cos 5x + \sin 2x = \cos 2x.\cot 3x$

$\Leftrightarrow \sqrt{2}\cos5x\sin 3x + \sin 2x\cos 3x = \cos 2x.\cos 3x$

$\Leftrightarrow \sqrt{2}\cos5x\sin 3x - \cos5x = 0 \Leftrightarrow \cos5x(\sqrt{2}\sin 3x - 1) = 0$

+) $\sin 3x = \frac{1}{\sqrt{2}} \neq 0$ (t/m đk) $\Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{12} + \frac{k2\pi}{3} \\ x = \frac{\pi}{4} + \frac{k2\pi}{3} \end{array}\right.$ +) $\cos5x = 0 \Leftrightarrow x = \frac{\pi}{10} + \frac{k\pi}{5}$ t/m đk

4) $\dfrac{6\sqrt{2}\sin^3 2x + 8\cos^3 x + 3\sqrt{2}\cos(\frac{17\pi}{2} - 4x)\cos 2x}{\cos x} = 16$ với $x \in (\frac{\pi}{2}; \frac{5\pi}{2})$

Ta có: $\cos x \neq 0 \Leftrightarrow x \neq \frac{\pi}{2} + k\pi$

Với đk pt(1) $\Leftrightarrow 8\cos^3 x + 6\sqrt{2}\sin 2x\left(\sin^2 2x + \cos^2 2x\right) = 16\cos x$

$\Leftrightarrow 4\cos^3 x + 3\sqrt{2}\sin 2x = 8\cos x \Leftrightarrow (2\cos^2 x + 3\sqrt{2}\sin x - 4) = 0$

$\Leftrightarrow 2\sin^2 x - 3\sqrt{2}\sin x + 2 = 0 \quad \Leftrightarrow x = \frac{\pi}{4} + k2\pi, x = \frac{3\pi}{4} + k2\pi\left(k \in \mathbb{Z}\right)$

Vậy phương trình đã cho có 2 nghiệm $x \in (\frac{\pi}{2}; \frac{5\pi}{2})$ là $x = \frac{3\pi}{4}; x = \frac{9\pi}{4}$

5) $\dfrac{\sqrt{3}}{\cos^2 x} + \dfrac{4}{\sin 2x} = 2(\cot x + \sqrt{3})$

Điều kiện $\sin 2x \neq 0 \Leftrightarrow x \neq \frac{k\pi}{2}$.

Ta có $\sqrt{3}\left(1 + \tan^2 x\right) + \dfrac{4}{\sin 2x} - 2\sqrt{3} = 2\cot x$

$\Leftrightarrow \sqrt{3}\tan^2 x + \dfrac{2(\sin^2 x + \cos^2 x)}{\sin x\cos x} - \sqrt{3} = 2\text{cotg } x$

$\Leftrightarrow \sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0$

$\tan x = -\sqrt{3} \Leftrightarrow x = -\frac{\pi}{3} + k\pi$ $\tan x = \frac{1}{\sqrt{3}} \Leftrightarrow x = \frac{\pi}{6} + k\pi$

6) Giải phương trình: $\cos2x + 2\sin x - 1 - 2\sin x\cos 2x = 0$ (1)

$\left(1\right) \Leftrightarrow \cos2x\left(1 - 2\sin x\right) - \left(1 - 2\sin x\right) = 0 \Leftrightarrow \left(\cos2x - 1\right)\left(1 - 2\sin x\right) = 0$

Khi cos2x = 1<=> $x = k\pi$, $k \in Z$

Khi $\sin x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6} + k2\pi$ hoặc $x = \frac{5\pi}{6} + k2\pi$, $k \in Z$

Xin chân thành cảm ơn quý thầy cô và các bạn học sinh đã đọc tài liệu này!

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